## Supplementary Problems Solutions for Exam 1

1. An object's initial velocity is $40 \mathrm{~m} / \mathrm{s}$ and is accelerating. When the object's displacement is 300 m , its velocity is $-30 \mathrm{~m} / \mathrm{s}$. What is the object's acceleration (in $\mathrm{m} / \mathrm{s}^{2}$ )?

$$
\begin{aligned}
\mathrm{v}^{2} & =\mathrm{v}_{\mathrm{o}}^{2}+2 \mathrm{ax} \\
(-30)^{2} & =40^{2}+2 \mathrm{a}(300) \\
\mathrm{a} & =1.17 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

2. A jet flying at $250 \mathrm{~m} / \mathrm{s}$ at an altitude of 900 meters, and climbing at an angle of $10^{\circ}$ relative to the horizontal, drops a bomb. What is the "line-of-sight" distance $d$ (in meters) between the bomb's initial location and the location on the ground where the bomb lands?


$$
\begin{aligned}
& y=v_{\text {oy }} t+1 / 2 a_{y} t^{2} \\
& -900=250\left(\sin 10^{\circ}\right) t+1 / 2(-9.8) t^{2} \\
& t=18.69 \mathrm{~s} \\
& \\
& x= \\
& =v_{\text {ox }} t+1 / 2 a_{x} t^{2} \\
& =250\left(\cos 10^{\circ}\right) 18.69+1 / 2(0) \mathrm{t}^{2} \\
& =4601.51 \mathrm{~m}
\end{aligned}
$$

Find the line-of-sight distance, d:

4601.51 m

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\(\mathrm{d}=\) hypotenuse of right triangle
    \(=\left(4601.51^{2}+900^{2}\right)^{1 / 2}\)
    \(=4688.7 \mathrm{~m}\)
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3. At time $t=0$ an automobile moving at $30 \mathrm{~m} / \mathrm{s}$ accelerates at the rate of $2 \mathrm{~m} / \mathrm{s}^{2}$. How far will the object travel during the time interval between $\mathrm{t}=14 \mathrm{~s}$ and $\mathrm{t}=16 \mathrm{~s}$ ?

$$
\begin{aligned}
\mathrm{x} & =\mathrm{v}_{\mathrm{o}} \mathrm{t}+1 / 2 \mathrm{at}^{2} \\
& =30(16)+1 / 2(2)(16)^{2} \\
& =736 \mathrm{~m} \\
\mathrm{x} & =30(14)+1 / 2(2)(14)^{2} \\
& =616 \mathrm{~m}
\end{aligned}
$$

$$
736-616=120 m
$$

4. A ball is thrown straight upward at $20 \mathrm{~m} / \mathrm{s}$ from the top of a tall building, reaches maximum height, then falls towards the ground. How many seconds after being thrown will its velocity be $-30 \mathrm{~m} / \mathrm{s}$ ?


$$
\begin{aligned}
v_{y} & =v_{o y}+a_{y} t \\
-30 & =20-9.8 \mathrm{t} \\
\mathrm{t} & =5.10 \mathrm{~s}
\end{aligned}
$$

5. A bullet is fired straight upward. How many seconds after it reaches its maximum height will it have fallen 40 meters?


We start the clock when the bullet has reached maximum height. From that point on, the flight of the bullet is the same as if it were dropped:

$$
\begin{aligned}
\mathrm{v}_{\mathrm{yo}} & =0 \\
\mathrm{y} & =\mathrm{v}_{\mathrm{yo}} \mathrm{t}+1 / 2 \mathrm{ay}^{2} \mathrm{t}^{2} \\
-40 & =0+1 / 2(-9.8) \mathrm{t}^{2} \\
\mathrm{t} & =2.86 \mathrm{~s}
\end{aligned}
$$

6. The velocity of an object changes from $120 \mathrm{~m} / \mathrm{s}$ to $70 \mathrm{~m} / \mathrm{s}$ in four seconds. What was the object's displacement (in meters) during this time?

$$
\begin{aligned}
x & =\text { average velocity } \mathrm{x} \text { time } \\
& =1 / 2(120+70)(4) \\
& =380 \mathrm{~m}
\end{aligned}
$$

7. A bullet is fired over level ground at $200 \mathrm{~m} / \mathrm{s}$ toward a cliff face 250 meters away. At what angle (in degrees) relative to the horizontal must the bullet be fired in order to strike the wall 60 meters above the point from which the bullet was fired?


$$
\begin{aligned}
x & =v_{o x} t+1 / 2 a_{x} t^{2} \\
250 & =200(\cos \theta) t+0 \\
t & =250 /(200 \cos \theta) \\
y & =v_{\text {oy }} t+1 / 2 a_{y} t^{2} \\
60 & =(200 \sin \theta) t+1 / 2(-9.8) t^{2} \\
& =(200 \sin \theta) 250 /(200 \cos \theta)-4.9[250 /(200 \cos \theta)]^{2}
\end{aligned}
$$

$\theta=15.26^{\circ}$
8. A stone is thrown from the top of a building 20 meters tall at a speed of $16 \mathrm{~m} / \mathrm{s}$ and at an angle of 30 degrees below the horizontal. How far (in meters) horizontally will the stone have traveled when it strikes the ground?


$$
\begin{aligned}
\mathrm{y} & =\mathrm{v}_{\mathrm{oy}} \mathrm{t}+1 / 2 \mathrm{a}_{\mathrm{y}} \mathrm{t}^{2} \\
-20 & =(-16 \sin 30) \mathrm{t}-4.9 \mathrm{t}^{2} \\
\mathrm{t} & =1.363 \mathrm{~s} \\
\mathrm{x} & =v_{\mathrm{ox}} \mathrm{t}+1 / 2 \mathrm{ax}^{2} \mathrm{t}^{2} \\
& =(16 \cos 30)(1.363)+0 \\
= & 18.88 \mathrm{~m}
\end{aligned}
$$

9. A struck golf ball leaves the ground with speed $90 \mathrm{~m} / \mathrm{s}$, traveling initially along an angle of 40 degrees above the horizontal. How far (in meters) horizontally will it have traveled by the time its elevation is 30 meters for the second time?


$$
\begin{aligned}
& \mathrm{y}=\mathrm{v}_{\text {oy }} \mathrm{t}+1 / 2 \mathrm{ayy}^{2} \mathrm{t}^{2} \\
& 30=(90 \sin 40) \mathrm{t}+1 / 2(-9.8) \mathrm{t}^{2} \\
& \mathrm{t}=0.54 \mathrm{~s} \text { and } 11.26 \mathrm{~s} \\
& \mathrm{x}=v_{\text {ox }} \mathrm{t}+1 / 2 \mathrm{axx}^{2} \\
& =(90 \cos 40)(11.26)+0 \\
& =776 \mathrm{~m}
\end{aligned}
$$

10. The gravitational force two objects of equal mass exert on each other is 6000 N . What would be the new force (in newtons) if the separation is tripled, and the mass of each object is doubled?

$$
\begin{aligned}
\text { Old } \mathrm{F} & =\mathrm{Gm}_{1} \mathrm{~m}_{2} / \mathrm{r}^{2} \\
& =6000 \mathrm{~N}
\end{aligned}
$$

New F $=\mathrm{G}\left(2 \mathrm{~m}_{1}\right)\left(2 \mathrm{~m}_{2}\right) /(3 \mathrm{r})^{2}$
$=(4 / 9) \mathrm{Gm}_{1} \mathrm{~m}_{2} / \mathrm{r}^{2}$
$=(4 / 9)(6000)$
$=2667 \mathrm{~N}$
11. An object weighing 500 N is placed in an elevator that is accelerating. What would have to be the acceleration (in $\mathrm{m} / \mathrm{s}^{2}$ ) of the elevator in order that the object's apparent weight be only 100 N ?

$$
\begin{aligned}
\mathrm{C} & =100 \mathrm{~N} \\
\mathrm{~m} & =500 / 9.8 \\
& =51.02 \mathrm{~kg}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{F} & =\mathrm{ma} \\
-500+100 & =(51.02) \mathrm{a} \\
\mathrm{a} & =-7.84 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

The negative sign indicates the elevator is accelerating downward.
12. Three forces in a plane act on an object. Their magnitudes and headings $\theta$ are indicated in the list below:
$\mathrm{A}=200 \mathrm{~N} \quad \theta=40^{\circ}$
$\mathrm{B}=160 \mathrm{~N} \quad \theta=110^{\circ}$
$\mathrm{C}=300 \mathrm{~N} \quad \theta=200^{\circ}$

What is the heading (in degrees) of the net force?


| $\begin{aligned} \mathrm{A}_{\mathrm{x}} & =200 \sin 40^{\circ} \\ & =128.56 \mathrm{~N} \end{aligned}$ | $\begin{aligned} \mathrm{B}_{\mathrm{x}} & =160 \cos 20^{\circ} \\ & =150.35 \mathrm{~N} \end{aligned}$ | $\begin{aligned} \mathrm{C}_{\mathrm{x}} & =-200 \sin 60 \\ & =-173.21 \mathrm{~N} \end{aligned}$ |
| :---: | :---: | :---: |
| $\begin{aligned} \mathrm{A}_{\mathrm{y}} & =200 \cos 40^{\circ} \\ & =153.21 \mathrm{~N} \end{aligned}$ | $\begin{aligned} \mathrm{B}_{\mathrm{y}} & =-160 \sin 20^{\circ} \\ & =-54.72 \mathrm{~N} \end{aligned}$ | $\begin{aligned} C_{y} & =200 \cos 60 \\ & =100.00 \mathrm{~N} \end{aligned}$ |

$$
\begin{aligned}
& \hline \text { Call the net force: } \mathrm{F} \\
& \\
& \mathbf{F}=\mathbf{A}+\mathbf{B}+\mathbf{C} \\
& \mathrm{F}_{\mathrm{x}}=\mathrm{A}_{\mathrm{x}}+\mathrm{B}_{\mathrm{x}}+\mathrm{C}_{\mathrm{x}} \\
&=128.56+150.35-173.21 \\
&=105.70 \mathrm{~N} \\
& \mathrm{~F}_{\mathrm{y}}=\mathrm{A}_{\mathrm{y}}+\mathrm{B}_{\mathrm{y}}+\mathrm{C}_{\mathrm{y}} \\
&=153.21-54.72+100.00 \\
&=198.49 \mathrm{~N}
\end{aligned}
$$


13. An object of mass 50 kg is 5.0 m from an object of mass 120 kg . How far from the $50-\mathrm{kg}$ object, on the line connecting the two objects, would the net force on a third object of mass m be zero?


The sum of the two forces is zero:
$-G(50) m / x^{2}+G(120) m /(5-x)^{2}=0$
Divide by Gm:

$$
\begin{aligned}
-50 / x^{2}+120 /(5-x)^{2} & =0 \\
x & =1.96 \mathrm{~m}
\end{aligned}
$$

14. At what firing angle (in degrees) above the horizontal must an arrow, fired at $80 \mathrm{~m} / \mathrm{s}$, be aimed to rise to a maximum height of 170 meters?


Rise Time:
Velocity at the top is zero: $\mathrm{v}_{\mathrm{y}}=0$
$\mathrm{v}_{\mathrm{y}}=\mathrm{v}_{\mathrm{oy}}+\mathrm{a}_{\mathrm{y}} \mathrm{t}$
$0=v_{o} \sin \theta-g t$
$\mathrm{t}=\mathrm{v}_{\mathrm{o}} \sin \theta / \mathrm{g} \quad$ Equation 1
(This is the rise time.)
Average Velocity Rising:
$1 / 2\left(v_{o y}+v_{y}\right)=1 / 2\left(v_{o} \sin \theta+0\right)$
$=1 / 2 \mathrm{v}_{\mathrm{o}} \sin \theta$
$\mathrm{y}=($ average velocity rising ) (rise time)
$=\left(1 / 2 \mathrm{v}_{\mathrm{o}} \sin \theta\right)\left(\mathrm{v}_{\mathrm{o}} \sin \theta / \mathrm{g}\right)=1 / 2$
$=1 / 2\left(\mathrm{v}_{\mathrm{o}} \sin \theta\right)^{2} / \mathrm{g}$
$170=1 / 2(80 \sin \theta)^{2} / 9.8$
$\theta=46.18^{\circ}$
15. Two blocks are touching on a frictionless tabletop, as shown in the figure below. Two opposing horizontal forces are applied. What is the contact force (in newtons) each block exerts on the other?


Apply F = ma to the $9-\mathrm{kg}$ pair:

$$
\begin{aligned}
\mathrm{F} & =\text { Net Force } \\
& =76-40 \\
& =36 \mathrm{~N} \\
\mathrm{~F} & =\mathrm{ma} \\
36 & =9 \mathrm{a} \\
\mathrm{a} & =4 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Now, apply F = ma to the $5-\mathrm{kg}$ block:


$$
\begin{aligned}
\mathrm{F} & =\mathrm{ma} \\
\mathrm{C}-40 & =5(4) \\
\mathrm{C} & =60 \mathrm{~N}
\end{aligned}
$$

Or, we could have applied $\mathrm{F}=\mathrm{ma}$ to the $4-\mathrm{kg}$ block:


$$
\begin{aligned}
\mathrm{F} & =\mathrm{ma} \\
76-\mathrm{C} & =4(4) \\
\mathrm{C} & =60 \mathrm{~N}
\end{aligned}
$$

16. Two cars are initially separated by 860 meters. Car A is moving at $20 \mathrm{~m} / \mathrm{s}$ and accelerating at $4 \mathrm{~m} / \mathrm{s}^{2}$ and is chasing Car B, which is moving at a steady velocity of 30 $\mathrm{m} / \mathrm{s}$. How far (in meters) will Car A have traveled by the time it catches up with Car B?


Choose the "zero" on the x-axis to be Car A's location initially.

## Car A

$$
\begin{aligned}
\mathrm{x} & =\mathrm{x}_{\mathrm{o}}+v_{\mathrm{o}} \mathrm{t}+1 / 2 \mathrm{at}^{2} \\
& =0+20 \mathrm{t}+1 / 2(4) \mathrm{t}^{2} \\
& =20 \mathrm{t}+1 / 2(4) \mathrm{t}^{2}
\end{aligned}
$$

## Car B

$$
\begin{aligned}
x & =x_{0}+v_{o} t+1 / 2 \mathrm{at}^{2} \\
& =860+30 \mathrm{t}
\end{aligned}
$$

Car A will have caught up with Car B when the cars' $x$-coordinates are the same:

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\(20 t+1 / 2(4) t^{2}=860+30 t\)
\(\mathrm{t}=-18.39 \mathrm{~s}\)
and
\(\mathrm{t}=23.39 \mathrm{~s}\)
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We reject the negative time solution as physically meaningless.
Calculate the x-coordinate of Car A travels after it has been traveling 23.39 s :

$$
\begin{aligned}
\mathrm{x} & =20 \mathrm{t}+1 / 2(4) \mathrm{t}^{2} \\
& =20(23.39)+1 / 2(4)(23.39)^{2} \\
& =1562 \mathrm{~m}
\end{aligned}
$$

17. Two cars are initially separated by 1800 meters. Car A is moving at $30 \mathrm{~m} / \mathrm{s}$ and accelerating at $3.2 \mathrm{~m} / \mathrm{s}^{2}$ and is headed toward Car B, which is moving at $-20 \mathrm{~m} / \mathrm{s}$. After how many seconds will the cars collide?

Choose the "zero" reference location to be Car A's initial location.


## Car A

$$
\begin{aligned}
\mathrm{x} & =\mathrm{x}_{0}+\mathrm{v}_{\mathrm{o}} \mathrm{t}+1 / 2 \mathrm{at}^{2} \\
& =0+30 \mathrm{t}+1.6 \mathrm{t}^{2} \\
& =30 \mathrm{t}+1.6 \mathrm{t}^{2}
\end{aligned}
$$

## Car B

$$
\begin{aligned}
\mathrm{x} & =\mathrm{x}_{\mathrm{o}}+\mathrm{v}_{\mathrm{o}} \mathrm{t}+1 / 2 \mathrm{at}^{2} \\
& =1800-20 \mathrm{t}+0
\end{aligned}
$$

The cars collide when their x-coordinates are the same:

$$
\begin{aligned}
30 \mathrm{t}+1.6 \mathrm{t}^{2} & =1800-20 \mathrm{t} \\
\mathrm{t} & =21.38 \mathrm{~s}
\end{aligned}
$$

