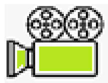
Physics 23 Notes Chapter 7

Impulse and Momentum

Dr. Joseph F. Alward

	Video Lecture: Overview

Impulse Symbol: I
Units: N-s

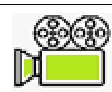
The "contact time" associated with a particular force is the amount of time the force is acting.

The "impulse" delivered by the force is the product of the average force and the contact time.

$$I = \bar{F}\,t$$

Units: N-s

Note for later use:
1.0 N-s = 1.0 (kg-m/s^2)-s
= 1.0 kg-m/s

	Boxer Absorbs Impulse

Example A:

The contact time is 80 milli-seconds during the collision between a bat and a baseball. During this time, the average force the bat exerts on the ball is 250 N.

What impulse did the ball received?

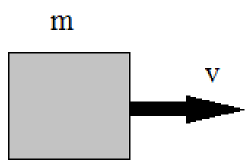
$I = \bar{F}\,t$
$= (250\text{ N})\,(80 \times 10^{-3}\text{ s})$
$= 20.0$ N-s

Example B:

Referring to the bat and ball example at the left, what impulse did the *bat* receive?

By Newton's 3rd Law, the magnitude of the contact force the bat experiences is the same as the contact force the ball experiences, and the contact times are obviously the same for each. Thus, the product $\bar{F}$ t is the same for each, but opposite in sign. The impulse the bat receives therefore is -20.0 N-s.

Linear Momentum



Momentum:
Symbol: p
Units: kg-m/s

$p = mv$

m = mass (in kilograms
v = velocity (in m/s)

Momentum is a vector whose direction is the same as the direction of the velocity vector: objects moving to the right have positive momentum, while objects moving to the left have negative momentum.

Example:

(a) An object of mass m = 50 kg is moving to the right at v = 30 m/s. What is its momentum?

p = 50 kg (30 m/s)
 = 1500 kg-m/s

(b) What would have been the object's momentum if it were moving to the *left* at -30 m/s?

p = 50 kg (-30 m/s)
 = -1500 kg-m/s

Impulse-Momentum Theorem

Prove that the total impulse delivered to an object equals its change in momentum.

Combine the two equations below:

$m\bar{a} = \bar{F}$ and $\bar{a} = (v - v_o)/t$

$$mv - mv_o = \bar{F}t$$

This is the Momentum-Impulse Theorem:

The change in momentum of an object equals the total impulse it received.

Other forms of this theorem are below:

$$p - p_o = I$$
$$\Delta p = I$$
$$\Delta p = \bar{F}t$$

Recall from page one of this chapter:

1 kg-m/s = 1 N-s

Thus, momentum units are equivalent to impulse units.

Example A:

A constant 40-N net force acts for 1.20 seconds on an object whose momentum initially was 42 kg-m/s. What is the new momentum?

In this case, the average force equals the constant force F = 40 N:

$$p - p_o = F\,t$$
$$p - 42 \text{ kg-m/s} = (40 \text{ N})(1.20 \text{ s})$$
$$= 48 \text{ N-s}$$
$$= 48 \text{ kg-m/s}$$
$$p = 90 \text{ kg-m/s}$$

Example B:

A ten-kilogram object moving to the right at 4.0 m/s is struck a blow from behind that delivers an impulse of 60 N-s. What is the resulting velocity of the object?

$$mv - mv_o = I$$
$$10\,v - 10\,(4.0) = 60$$
$$v = 10 \text{ m/s}$$

Example C:

In a two-millisecond period of time the momentum of an object decreased by 0.08 kg-m/s. What average total force caused this change?

$$\Delta p = \overline{F}\,t$$

$$-0.08 = \overline{F}\,(2.0 \times 10^{-3})$$

$$\overline{F} = -\,40 \text{ N}$$

The negative sign indicates that the average force was directed to the left.

Force vs Time Graph

<u>Example</u>:

An object moves under the influence of a net force F as shown in the graph. At time t = 0 the momentum of the object is 30 kg-m/s. What is its momentum at time t = 20 milli-seconds (ms)?

Using the equation

Use the momentum-impulse theorem to calculate the sum of the momentum changes for each of the four time intervals in the graph.

$$\Delta p = \overline{F}\, t$$

A: I = ½ (0 + 2000) (5)
  = 5,000 N-ms

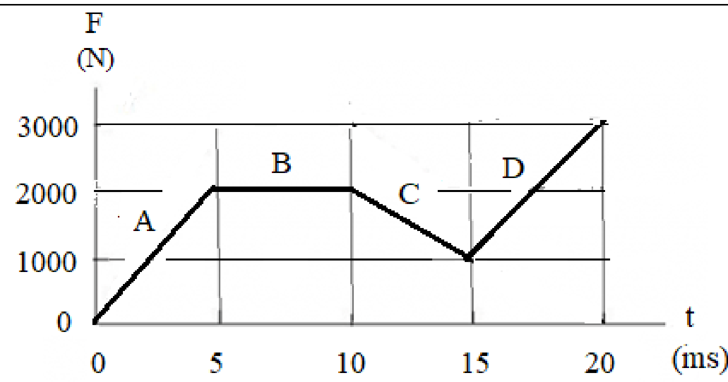
B: I = (2000)(5)
  =10,000 N-ms

C: I = ½ (1000 + 2000) (5)
  = 7,500 N-ms

D: I = ½ (1000 + 3000) (5)
  = 10,000 N-ms

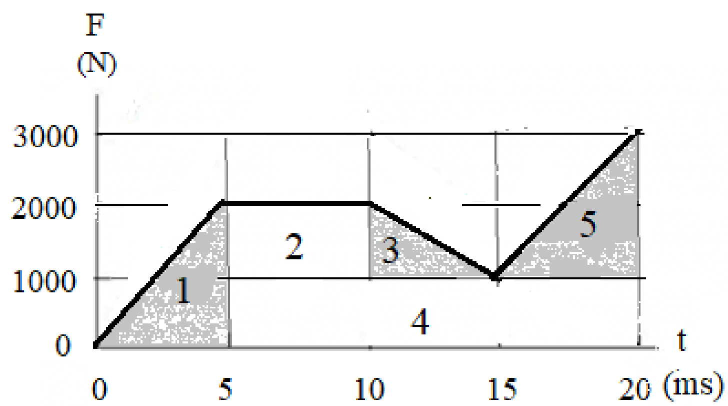
Sum = 32,500 N-ms
  = 32.5 N-s
  = 32.5 kg-m/s

$p - p_o = I$
$p = p_o + I$
  = 30.0 + 32.5
  = 62.5 kg-m/s



<u>Alternative Graphical Analysis</u>

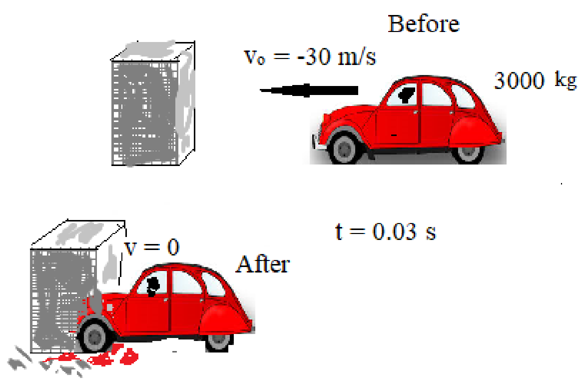
The impulse delivered by a force can be found by calculating the area under the force-time curve. The total area below is the sum of the areas of triangles 1, 3, and 5, and rectangular areas 2 and 4.



1: ½ (5) (2000) = 5000 N-ms
2. (5) (1000) = 5000 N-ms
3. ½ (5) 1000 = 2500 N-ms
4. (15) (1000) = 15,000 N-ms
5. ½ (5) (2000) = 5000 N-ms
Total: 32,500 N-ms = 32.5 N-s

Example:

A 3000-kg automobile moving to the left at -30 m/s crashes into a wall and comes to rest after 0.03 second.



(a) What average force did the wall exert on the car?

$$\bar{F}\, t = mv - mv_o$$
$$\bar{F}\,(0.03) = (3000)(0) - (3000)(-30)$$
$$\bar{F} = 3 \times 10^6 \text{ N}$$

This force is positive, so it's directed to the right.

(b) What average force did the car exert on the wall?

By Newton's 3rd Law, the car exerts a force equal but *opposite* on the wall:

$$\bar{F} = -3 \times 10^6 \text{ N}$$

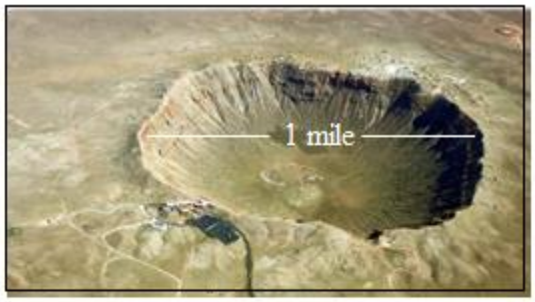
The sign of this force is negative, so the force the car exerts on the wall is directed to the left.

Example:

Approximately 50,000 years ago, a meteorite having a mass of about 5×10^8 kg* collided with Earth at an estimated speed of 1.28×10^4 m/s and came to rest in about 2.0 seconds in what is now the northern Arizona desert. A crater about one mile across and 600 feet deep was created.

What average force did the meteorite exert on Earth?

Answer: By Newton's Third Law, the force the meteorite exerted on Earth is the opposite of the force Earth exerted on the meteorite.



$$\bar{F}\, t = mv - mv_o$$
$$\bar{F}(2.0) = 0 - 5 \times 10^8\, (-1.28 \times 10^4)$$
$$\bar{F} = 3.2 \times 10^{12}\text{ N}$$ (This is the force Earth exerts.)

(Three trillion newtons is about 700 million pounds, which is the weight of about 200 thousand automobiles).

Note that the positive force above is the upward force the ground exerted on the asteroid.

For comparison, the asteroid that smashed into the Yucatan peninsula 65 million years ago had a mass more than *one hundred million* times greater than the one that left the crater above. The Yucatan asteroid left a crater about 50 miles in diameter.

* An average automobile has a mass of 1000 kg. So the asteroid had the mass of 5000 automobiles:

5×10^8 kg/ 1000 = 5,000

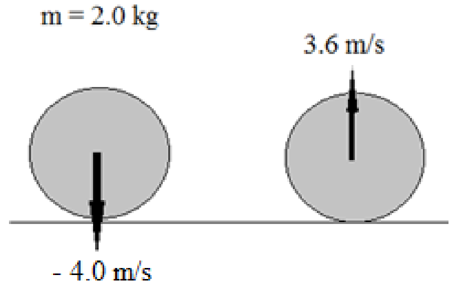
Example A:

The ball in the figure at the right strikes the floor moving downward at a velocity of -4.0 m/s, and rebounds upward with a velocity of 3.6 m/s.

The ball was in contact with the floor for 0.10 second.

During the contact time period there was a varying contact force, as well as a constant weight force mg acting on the ball.

$$\begin{aligned} \bar{F} &= \bar{C} - mg \\ &= \bar{C} - 2\,(9.8) \\ &= \bar{C} - 19.6 \end{aligned}$$



What was the average contact force?

$$\begin{aligned} \bar{F}\,t &= mv - mv_o \\ [\,\bar{C} - 19.6\,]\;0.10 &= 2.0\,(3.6) - 2.0(-4.0) \\ \bar{C} &= 171.60\text{ N} \end{aligned}$$

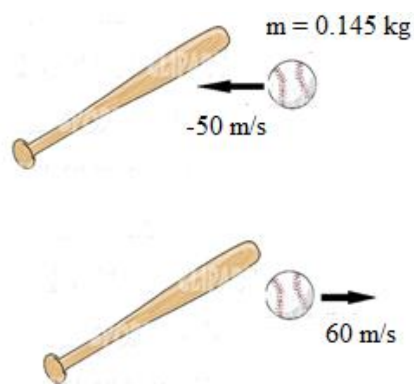
Example B:

A 0.145 kg baseball traveling to the left with velocity v_o = -50 m/s is struck by a bat, and is in contact with the bat for 2.0 milli-seconds.

After losing contact with the bat, the ball's velocity is v = 60 m/s.

What average force does the bat exert on the ball?

$$\begin{aligned} \bar{F}\,t &= mv - mv_o \\ \bar{F}(2.0 \times 10^{-3}) &= 0.145\,(60) - (0.145)\,(-50) \\ \bar{F} &= 7975\text{ N} \\ &= 1792\text{ lbs} \end{aligned}$$



Follow-Through

A thrown ball starts out at rest in the hand of the thrower, and leaves the hand of the thrower at a certain speed, v.

Use the impulse-momentum theorem to obtain the relationship between the speed at which the ball leaves the hand, amount of time the hand was in contact with the ball, and the average force the hand exerts on the ball.

$$
\begin{aligned}
\bar{F}t &= mv - mv_o \\
&= mv - m(0) \\
\bar{F}t &= mv \\
v &= \bar{F}\,t/m \\
&= (\bar{F}/m)t
\end{aligned}
$$

Leaning into the throw prolongs the contact time, which increases v, which increases distance.

In the previous example, we obtained the equation below:

$$v = (\bar{F}/m)\,t$$

If a bat, golf club, tennis racket, or a throwing arm, is swung in a manner that prolongs contact with the ball, this “follow-through” maximizes contact time, which likewise maximizes the ball’s speed v and the distance the struck object travels.

Another Impulse Example

A baseball leaves the hand of the thrower with speed $v = 30$ m/s:

$$v = \bar{F}\, t/m$$
$$\bar{F}\, t/m = 30 \text{ m/s}$$

(a) What would have been the speed v if the contact time had been 15% greater, and the average force 20% larger?

$$\text{New } v = (\text{New } \bar{F})\,(\text{New } t)/m$$
$$= (1.20\, \bar{F})\,(1.15\, t)/m = 1.38\,(\bar{F}\, t/m)$$
$$= 1.38\,(30)$$
$$= 41.40 \text{ m/s}$$

(b) By what multiple would the distance of travel be increased if these changes were made?

Recall from the Chapter 3 dealing with projectile motion that the range of a projectile traveling over level ground is

$$R = v_o^2 \sin(2\theta)/g$$

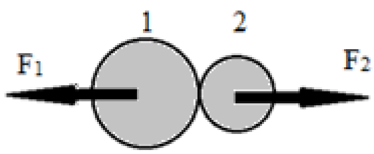
Increasing v_o to $1.38\, v_o$ will increase the range to $1.38^2 = 1.90$ times as great.

Conservation of Momentum in One Dimension

We now consider objects moving along the same straight line (in "one-dimensional motion") before and after they collide. As before, objects moving to the right have positive velocities, while those moving to the left
have negative velocities.

Two objects, numbered 1 and 2, collide.

F_1 = Force on Object 1 by Object 2
F_2 = Force on Object 2 by Object 1



By Newton's Third Law:
$\overline{F}_1 = -\overline{F}_2$

$\overline{F}_1 + \overline{F}_2 = 0$

Multiply by the contact time:

$\overline{F}_1 t + \overline{F}_2 t = 0$
Apply the Impulse-Momentum Theorem:
$\Delta p_1 + \Delta p_2 = 0$
$\Delta p_1 = - \Delta p_2$

<u>Gain-Loss Principle</u>

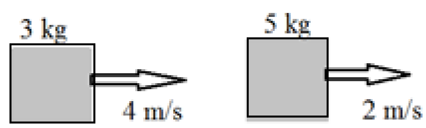
The equation above expresses the fact that the amount of momentum lost by one object equals the gain by the other one. Therefore, the total momentum *after* collision is the same as the total momentum *before* collision:

$$P = P_o$$

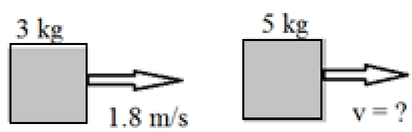
(We use <u>upper</u>-case "P" to represent the sum of the two momentums.)

Example:

Determine the speed of the 5-kg object below after collision.



Before Collision



After Collision

$$P = P_o$$
$$3(1.8) + 5\,v = 3(4) + 5(2)$$
$$v = 3.32 \text{ m/s}$$

Note: The velocity v is positive, as expected, because we expect the 5-kg object that's already moving to the right, and then is rear-ended, will continue moving to the right after collision.

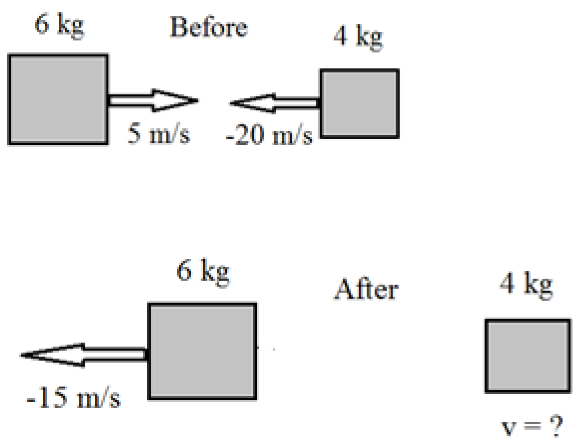
As a check on our work, show that the 3-kg object loses as much momentum as the 5-kg object gains:

3-kg: $\Delta p = 3(1.8) - 3\,(4)$
$= -6.60$ kg-m/s

5-kg: $\Delta p = 5\,(3.32) - 5\,(2)$
$= 6.60$ kg-m/s

Example A:

What is the velocity of the 4-kg object below after collision?



$$P = P_o$$
$$6\ (-15) + 4\ v = 6\ (5) + 4\ (-20)$$
$$v = 10 \text{ m/s}$$

Check Results: The 4-kg object gains120 kg-m/s of momentum, while the 6-kg object loses 120 kg-m/s of momentum.

Example B:

A 70-gram (0.070 kg) arrow is fired at 90 m/s to the right by a 60-kg archer at an angle of 40^o above the horizontal.

Assuming the archer is standing on a frictionless surface, what velocity does he acquire?

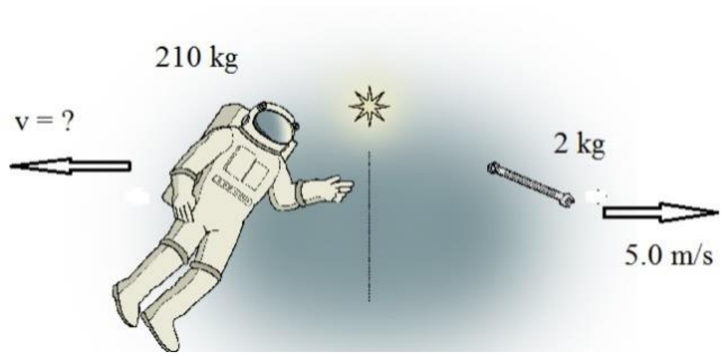
The arrow and the archer are each initially at rest, so the initial total x-momentum is zero, which means the later total x-momentum is likewise zero:

$$P = P_o$$
$$60\ v + 0.070\ (90) \cos 40 = 0$$
$$v = -0.08 \text{ m/s}$$

Negative sign indicates that the archer is sliding to the left after firing the arrow.

Example:

A 2-kg wrench in the hand of a 210-kg spacewalker initially at rest is thrown to the right at a velocity 5.0 m/s. What velocity does the spacewalker acquire?

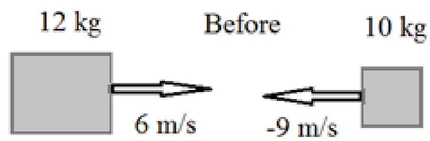


The wrench and the person holding it were each at rest initially: $P_o = 0$

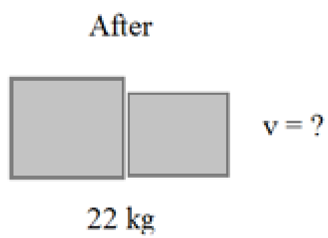
$$P = P_o$$
$$2.0\,(5.0) + 210\,v = 0$$
$$v = -0.048 \text{ m/s} \quad \text{(to the left)}$$

<u>Example</u>:

The objects below collide and stick together. What is the velocity of the pair after collision?



$P_o = 12\ (6) + 10\ (-9)$
$= -18$ kg-m/s



$P = 22\ v$

$P = P_o$
$22\ v = -18$
$v = -0.82$ m/s

Elastic Collisions

If the total kinetic energy of colliding objects is conserved, the collision is "elastic." If it's not conserved, the collision is called "inelastic."

Elastic collisions are rare. They occur if none of the kinetic energy is used to do work to permanently deform the object, create sound, or heat. Collisions of nuclei, for example, are elastic. Collisions of highest-quality steel balls are nearly elastic.

If a rubber ball were dropped to the ground and it rebounded to the same height from which is was dropped, then the collision would be elastic. Ignoring air resistance, the ball would never stop bouncing.

During elastic collisions, two laws are obeyed:

1. Total momentum is conserved. (Total momentum is conserved in *all* types of collisions.)

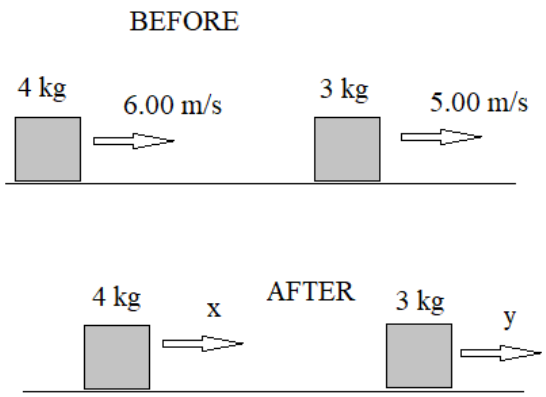
2. Total kinetic energy is conserved.

If the collision is not elastic, only momentum is conserved.

On examinations, students will be specifically told if the collision is elastic; if the problem doesn't state that the collision is elastic, students should assume the collision is *not* elastic, so only momentum is conserved.

Example:

The collision below is elastic.



Find the velocities x, and y.

Conserve Momentum:

$$\mathbf{P = P_o}$$
$$4x + 3y = 4(6) + 3(5)$$
$$y = (39 - 4x)/3 \qquad \text{(Equation 1)}$$

Conserve Kinetic Energy:

$$\mathbf{K = K_o}$$
$$½ (4)\, x^2 + ½ (3)\, y^2 = ½ (4)\, 7^2 + ½ (3)\, 5^2$$
$$4x^2 + 3y^2 = 219$$

Replace y by the expression in Equation 1:

$$4x^2 + 3\,[(39 - 4x)/3]^2 = 219$$

x = 5.14 m/s and y = 6.14 m/s
or
x = 6.00 m/s and y = 5.00 m/s

Reject the solution pair, x = 6.00 m/s and y = 5.00 m/s as nonsense. These velocities would mean that the 4-kg object supernaturally passes through the 3-kg object without either velocity changing; this cannot happen.

Inelastic Collisions in Two Dimensions

In collisions in which the objects are moving before and after in the same plane, such as colliding billiard balls on a pool table, the x- and y-components of the total momentum are conserved. We symbolize the total momentum--the sum of the two separate momentums--with upper-case "P."

$$P_x = P_{ox}$$
$$P_y = P_{oy}$$

<u>Example</u>:

Determine the direction **θ** and speed **v** of the 15-kg object after collision.

$$P_x = P_{ox}$$
$$15\ v \cos\theta + 20(70) \cos 30 = 15\ (90) \sin 50 + 20\ (60) \sin 70$$

$$v \cos\theta = 63.29 \text{ m/s} \qquad \text{Equation (1)}$$

$$P_y = P_{oy}$$
$$15\ v \sin\theta - 20\ (70) \sin 30 = -15\ (90) \cos 50 + 20\ (60) \cos 70$$

$$v \sin\theta = 16.18 \text{ m/s} \qquad \text{Equation (2)}$$

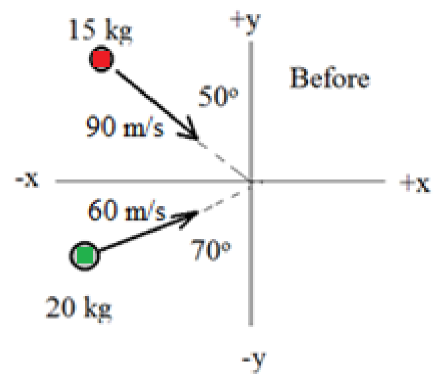
Divide Equation 2 by Equation 1:

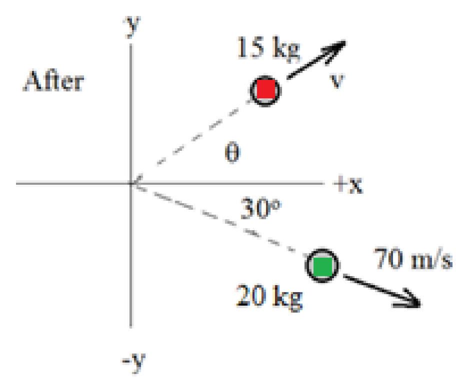
$$\tan\theta = 0.256$$
$$\boldsymbol{\theta = 14.36^o}$$

Substitute the angle θ either into Equation 1 (or Equation 2):

$$v \cos(14.36) = 63.29$$
$$\mathbf{v = 65.33\ m/s}$$

Your answers may differ slightly, depending on round-off.

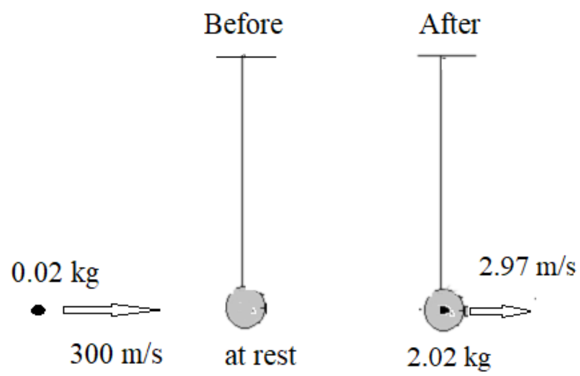




Example:

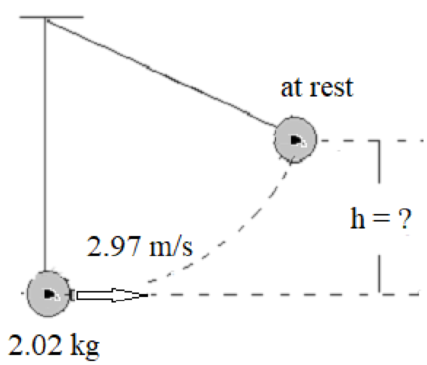
A 0.020-kg bullet is fired at 300 m/s into a 2.00-kg hanging object at rest and lodges inside; the ball/bullet rises to a certain height h before coming to a momentary rest.

(a) Conserve x-momentum to find the speed of the object immediately after the bullet came to rest inside of it.



$$P_x = P_{ox}$$
$$(2.02)V = 0.02\,(300)$$
$$v = 2.97 \text{ m/s}$$

(b) Conserve energy to find the height h to which the object rises.

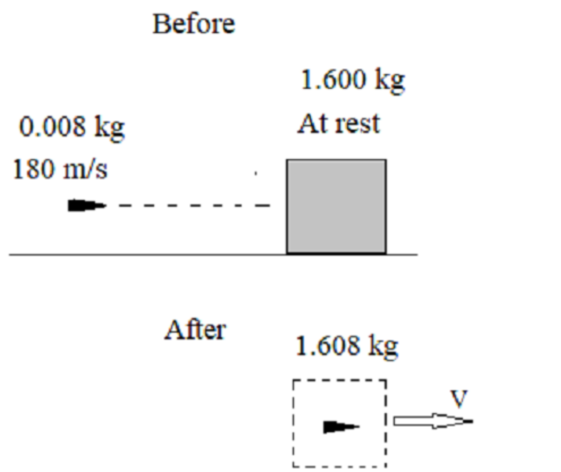


$$\tfrac{1}{2}\,mv^2 + mgh = \tfrac{1}{2}\,mv_o^2 + mgh_o$$
$$\tfrac{1}{2}\,v^2 + gh = \tfrac{1}{2}\,v_o^2 + gh_o$$
$$\tfrac{1}{2}\,(0)^2 + (9.8)\,h = \tfrac{1}{2}\,(2.97)^2 + (9.8)(0)$$
$$h = 0.45 \text{ m}$$

Example:

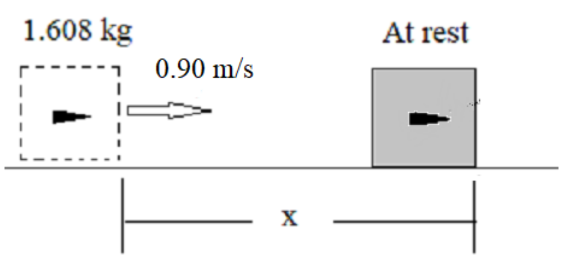
A bullet whose mass is 0.008 kg, moving at 180 m/s, collides with a block at rest whose mass is 1.600 kg, and is embedded in it immediately. The block with bullet inside slides along a horizontal surface. The coefficient of kinetic friction between the block and the surface is 0.03.

(a) Conserve momentum to find the velocity V of the block immediately after the bullet strikes it and comes to rest in it:



$$1.608\ v = 0.008\ (180)$$
$$v = 0.90\ \text{m/s}$$

(b) Use the work-kinetic energy theorem to find how far the block travels before coming to rest.



The contact force between the block and table is equal to the weight of the block:

$C = mg$
$= 1.608\ (9.8)$
$= 15.76$ N
$f = \mu C$
$= 0.03\ (15.76)$
$= 0.47$ N

The only unbalanced force acting is the frictional force, so the total force is F = 0.47 N.

Total Work = Change in kinetic energy

$$Fx \cos\theta = K - K_o$$
$$0.47\ x \cos 180 = 0 - ½\ (1.608)(0.90)^2$$
$$x = 1.39\ \text{m}$$