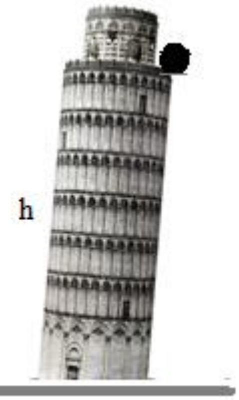
Physics 23 Chapter 6   Part Two

Conservation of Energy

Dr. Joseph F. Alward

Video Lecture: Summary of Important Content of Chapter 6.2



A boulder atop a tower, resting precariously at the edge, has a future capability of doing work, or a "potential" for doing work. The future work is the work that will be done after the boulder is tipped over the edge, crashes into the ground below and pushes the soil downward, forming a crater.

The "gravitational potential energy" of the boulder at a height h above the ground is symbolized as U, and is calculated as shown below:

$U = mgh$

The work a person would have to do to push an object of weight mg a distance h upward is given by the work equation:

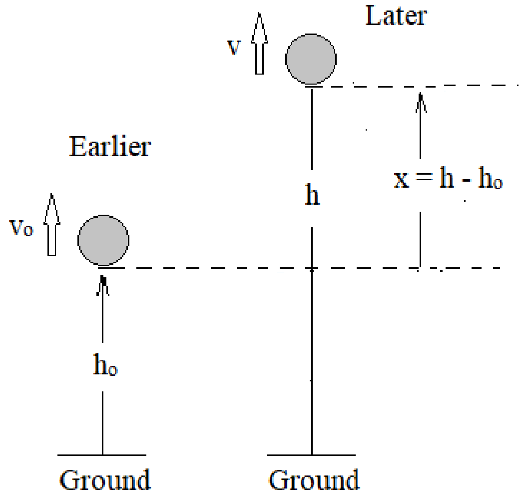
$$\begin{aligned} W &= Fx \cos \theta \\ &= mg\, h \cos 0 \\ &= mgh \end{aligned}$$

Thus, the potential energy of the boulder-tower "system" is equal to the work that was done to configure the system.

The Law of Conservation of Energy

Consider an object moving upward only under the influence of Earth's pull.

Initially, the object was at height h_o above the ground, and moving upward at speed v_o. Later, it's at height h moving at speed v.



Apply the equation below to the motion described at the left:

$$v^2 = v_o^2 + 2ax$$
$$= v_o^2 + 2\,(-g)\,(h - h_o)$$

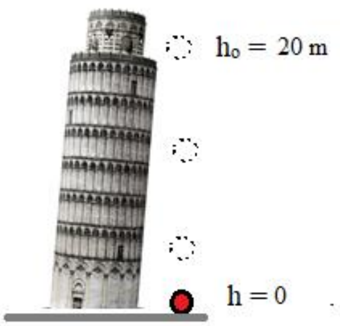
Multiple both sides by ½ m and re-arrange:

$$\tfrac{1}{2} mv^2 + mgh = \tfrac{1}{2} mv_o^2 + mgh_o$$
$$K + U = K_o + U_o$$
$$E = E_o$$

The total energy E at any moment is the same as its energy E_o at any earlier moment, i.e., total energy is *conserved*.

Choosing a Reference Location

<u>Example A</u>:



A 10-kg ball located atop the Leaning Tower of Pisa is dropped ($v_o = 0$) and lands on the ground.

h_o = 20 m
h = 0 m

What will be its kinetic energy when it strikes the ground ?

$\frac{1}{2} mv^2 + mgh = \frac{1}{2} mv_o^2 + mgh_o$

$\frac{1}{2} mv^2 = \frac{1}{2} mv_o^2 + mgh_o - mgh$
$= 0 + (10)(9.8)(20) - (10)(9.8)(0)$
$= 1960$ J

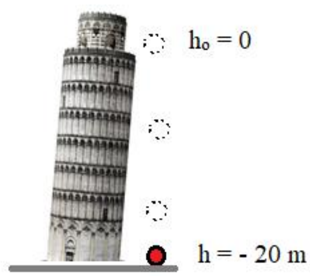
<u>Example B</u>:

In the tower example at the left, the literal ground is the location above which the heights of the falling ball are determined.

In problems involving the conservation of energy, *calculations of future velocities and kinetic energies <u>do not depend</u> on the choice of the reference location*.

To demonstrate this fact, let's calculate the ball's kinetic energy when it lands on the ground using a different reference location.

Let the reference location be the top of the tower:



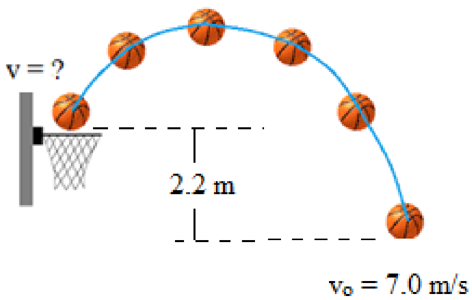
$\frac{1}{2} mv^2 + mgh = \frac{1}{2} mv_o^2 + mgh_o$

$\frac{1}{2} mv^2 = \frac{1}{2} mv_o^2 + mgh_o - mgh$
$= 0 + (10)(9.8)(0) - (10)(9.8)(-20)$
$= 1960$ J

This is the same result obtained in Example A, where a different reference location was used.

Students are free to choose whatever reference location they wish to solve conservation of energy problems.

Example A:



A basketball is thrown at speed 7.0 m/s, and is about to fall into the basket 2.2 meters above. What is its speed when it reaches the basket?

Let the point at which the ball was thrown be the reference location.

$h_o = 0$
$h = 2.2$ m
$v_o = 7.0$ m/s

$\frac{1}{2} mv^2 = \frac{1}{2} m (7.0)^2 + 0 - m (9.8)(2.2)$
Divide out the mass and solve for v:

$v = 2.42$ m/s

Example B:

After solving the problem at the left, solve it again, but this time use the basket's rim as the zero-reference elevation. Note that locations below the rim have negative heights.

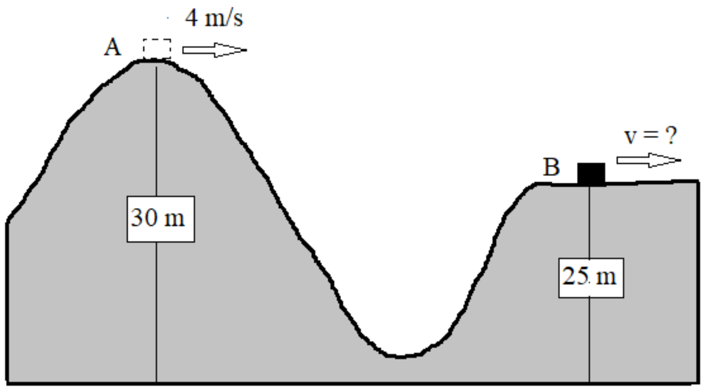
$h = 0$
$h_o = -2.2$ m
$v_o = 7.0$ m/s

$\frac{1}{2} mv^2 = \frac{1}{2} mv_o^2 + mgh_o - mgh$
$= \frac{1}{2} m (7.0)^2 + mg (-2.2) + 0$
$v = 2.42$ m/s

This answer is the same as the one obtained at the left.

One's choice of a reference elevation in any conservation of energy problem will not affect the ultimate answer.

Example:



A cart at Point A, 30 meters above the ground, is initially moving at 4.0 m/s on a frictionless roller-coaster. What will be its speed when it reaches point B, 25 m above the ground?

Let the ground be the reference location:

$$h_o = 30 \text{ m}$$
$$h = 25 \text{ m}$$
$$\tfrac{1}{2} mv^2 + mgh = \tfrac{1}{2} mv_o^2 + mgh_o$$
$$\tfrac{1}{2} mv^2 + m\,(9.8)(25) = \tfrac{1}{2} m\,(4.0)^2 + m\,(9.8)(30)$$

Divide both sides of the equation by m and solve for v:

$$\mathbf{v = 10.68\ m/s}$$

Repeat the calculation using Point B as the reference level:

$$h_o = 5 \text{ m}$$
$$h = 0$$
$$\tfrac{1}{2} mv^2 + mgh = \tfrac{1}{2} mv_o^2 + mgh_o$$
$$\tfrac{1}{2} mv^2 + m\,(9.8)(0) = \tfrac{1}{2} m\,(4.0)^2 + m\,(9.8)(5)$$
$$\mathbf{v = 10.68\ m/s}$$

Note: Our choice of a reference location at Point B makes the equation above simpler than it would have been: With our choice, the later potential energy is zero, so that's one less term to calculate.

Energy Gain-Loss Relationship

One of the forms of the energy conservation law is

$$K + U = K_o + U_o$$

Other forms are below:

$$(K - K_o) + (U - U_o) = 0$$
$$\Delta K + \Delta U = 0$$
$$\Delta K = - \Delta U$$
$$\Delta U = - \Delta K$$

The latter two equations express the fact that *the gain by one type of energy is the loss by the other.*

For example, if the potential energy decreases by 600 J, then the kinetic energy *increases* by 600 J.

<u>Example</u>:

The potential energy of a 20-kg object decreases by 400 joules; its final speed is 12 m/s. What was the object's initial speed?

Recall: The loss of one is gain of the other:

$$\Delta U = -400 \text{ J}$$
$$\Delta K = 400 \text{ J}$$
$$K - K_o = 400$$
$$½ (20)(12)^2 - ½ (20)v_o^2 = 400$$
$$v_o = 10.20 \text{ m/s}$$

Non-Conservative Work

When friction or air resistance exist, heat energy leaves the system, and the total energy is thereby decreased, i.e., no longer conserved: The later total energy E is less than the initial total energy E_o.

Forces such as friction and air resistance are called "non-conservative" forces and the work done by them is called "non-conservative" work, symbolized W_{NC}.

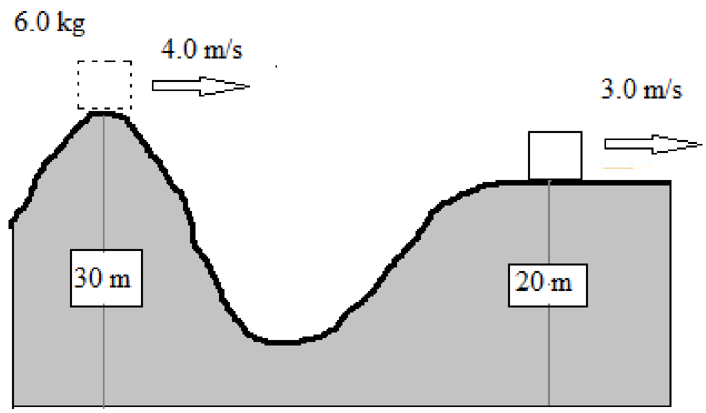
The later total energy is found using the equation below:

$E = E_o + W_{NC}$

<u>Example</u>:

$E_o = 700$ J $\qquad W_{NC} = -300$ J
$E = ?$
$E = E_o + W_{NC}$
$E = 700 + (-300)$
$= 400$ J

Example:



How much non-conservative work was done on the object in the figure above?

$E = ½ (6.0)(3.0)^2 + 6.0 (9.8)(20)$
$= 1203$ J

$E_o = ½ (6.0)(4.0)^2 + 6.0 (9.8)(30)$
$= 1812$ J

$E = E_o + W_{NC}$
$W_{NC} = E - E_o$
$= 1203 - 1812$
$= -609$ J

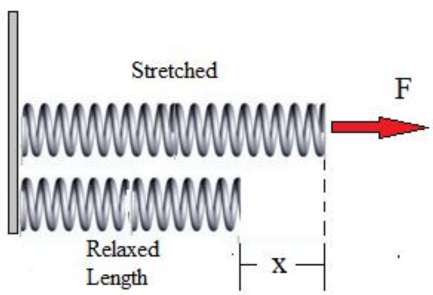
Note: Unlike all of the previous problems dealing with gravitational energy, in which the mass of the object played no role in obtain the answer to the question, in this case the mass matters. The reason for this is that the W_{NC} term contains no factor of m that could be divided away with the rest of the m's in the other parts of the equation.

Springs and Spring Potential Energy

The spring constant is a measure of a spring's stiffness. The spring constant is the magnitude of the force that must be applied for each meter of change in the length of the spring.

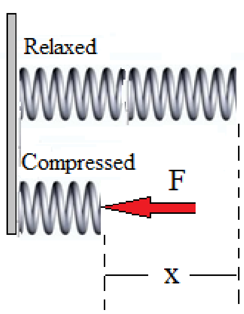
F = force magnitude

k = spring constant
Units: N/m



$$k = F/x$$

x = amount of stretching
(a positive number)



x = amount of compression
(a positive number)

<u>Example A</u>:

A 200-N force is required to stretch or compress--a spring by 0.40 m.

What is the spring constant?

$k = F/x$
$= 200$ N/0.40 m
$= 500$ N/m

....

<u>Example B</u>:

How much force is necessary to stretch a 900 N/m spring by 0.20 m?

$F = kx$
$= 900\ (0.20)$
$= 180$ N

....

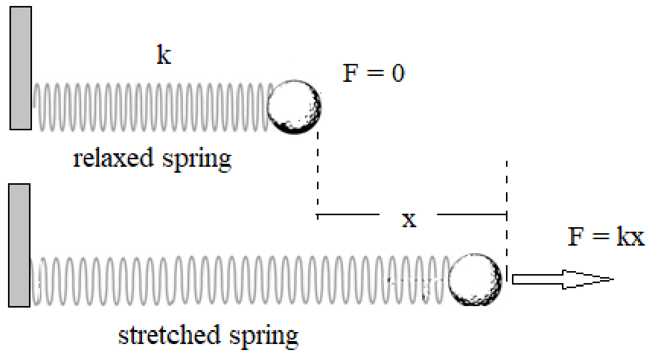
<u>Example C</u>:

During the stretching of a spring, the applied force increased from zero to 2500 N. What average force was applied?

Spring force varies linearly, so we may use the middle-value rule to determine the average force:

$\bar{F} = ½\ (0 + 2500)$
$= 1250$ N

Spring Potential Energy



$W = \bar{F} x \cos\theta$

$= \frac{1}{2}(0 + kx)\, x \cos 0^{o}$

$= \frac{1}{2} kx^2$

The work done to stretch the spring is ½ kx^2. We invoke the rule established earlier involving gravitational potential energy that states that the potential energy of a system equals the work done to configure it, so by this rule, the “spring potential energy” equals the work that is done to stretch the spring:

$U = \frac{1}{2} kx^2$

The equation for the spring potential energy of a compressed spring is the same:

$U = \frac{1}{2} kx^2$

Relaxed: $x = 0$
$U = 0$

Compressed by x:
$U = \frac{1}{2} kx^2$

Stretched by x:
$U = \frac{1}{2} kx^2$

Example:

A spring has a spring constant of 2000 N/m.

(a) What is its potential energy when it's stretched by 0.40 m?

$U = \frac{1}{2}(2000)(0.40)^2$
$= 160$ J

....

(b) What is its potential energy if it's *compressed* by 0.40 m?

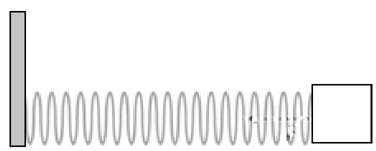
$U = \frac{1}{2}(2000)(0.40)^2$
$= 160$ J

Spring potential energy is "symmetrical" in the sense that compression and stretching store the same energies for the same x.

Spring-Mass Systems

A spring-mass "system" consists of an object and a spring.

<u>Example</u>:



A 20-kg object connected to a spring is oscillating on a frictionless tabletop. At a particular moment, the energies of the system are as listed below:

$K_o = 500$ J
$U_o = 900$ J

....

(a) What is the total energy of the system?

$E_o = K_o + U_o$
$= 500 + 900$
$= 1400$ J

....

(b) What is the kinetic energy when the potential energy is 300 J?

$K + U = E_o$
$K + 300 = 1400$
$K = 1100$ J

....

(c) Given that the object's mass is 20 kg, what is its speed when the potential energy is 130 J?

$K = 1400\text{ J} - 130\text{ J}$
$\frac{1}{2}(20)v^2 = 1270$
$v = 11.27$ m/s

In the following example of energy conservation there are *two* kinds of potential energy:

1. Gravitational potential energy: U^{grav}
2. Spring potential energy: U^{spring}

<u>Example</u>:

The object in the figure is held at the end of a relaxed spring, then released. How far downward (d = ?) does it fall before coming to a momentary rest?

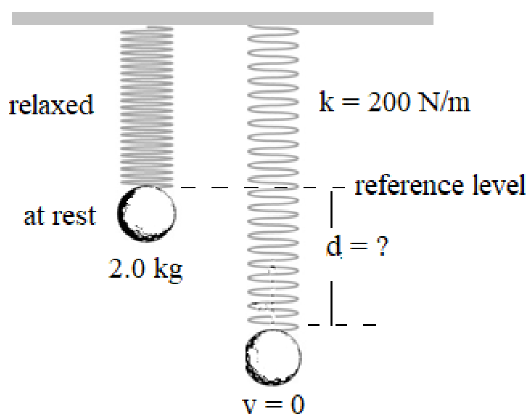
Choose the gravitational reference level to be at the bottom of the *relaxed* spring.

$h_o = 0$
$h = -d$

$E_o = K_o + U_o^{grav} + U_o^{spring}$
$\quad = ½\, mv_o^2 + mgh_o + ½\, kx_o^2$
$\quad = ½\, m(0)^2 + mg\,(0) + ½\, k(0)^2$
$\quad = 0$

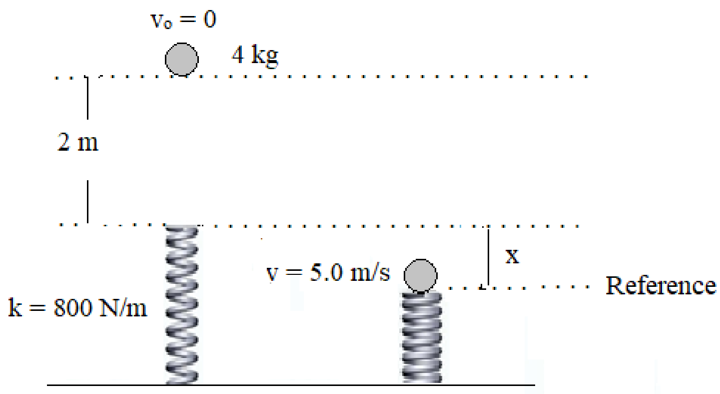
$E = K + U^{grav} + U^{spring}$
$\quad = ½\, mv^2 + mgh + ½\, kx^2$
$\quad = ½\, m(0)^2 + mg(-d) + ½\, kd^2$
$\quad = -mgd + ½\, kd^2$

$$E = E_o$$
$$-mgd + ½\, kd^2 = 0$$
$$-mg + ½\, kd = 0$$
$$-(2.0)(9.8) + ½\,(200)d = 0$$
$$d = 0.20 \text{ m}$$



Example:

A 4-kg object is dropped from rest toward a spring two meters below. By how much will the spring be compressed when the object's speed is 5.0 m/s?



Arbitrarily choose the top of the *compressed* spring as the reference elevation.

E_o = ½ (4) $(0)^2$ + 4 (9.8) (2 + x) + ½ (800) $(0)^2$
= 78.4 + 39.2 x

E = ½ (4)$(5)^2$ + mg (0) + ½ (800) x^2

Total Energy is Conserved:

E_o = E
78.4 + 39.2 x = ½ (4)$(5)^2$ + mg (0) + ½ (800) x^2
x = 0.32 m
and x = -0.22 m

The quantity x in all spring problems is the amount by which the spring is compressed or stretched, and is always a positive number. We therefore reject the negative answer above.

x = 0.32 m