Physics 23 Chapter 6 Part One

Dr. Joseph F. Alward

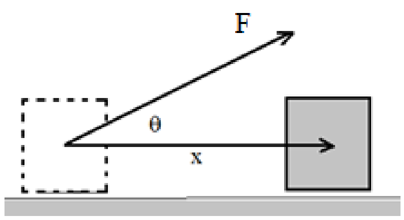
Work, Power, Kinetic Energy

	Video Lecture 1: Introduction to Work Video Lecture 2: Work as Area Video Lecture 3: Work Example

Work Done by Constant Forces

Work Symbol: W

In the figure below, a *constant* force is applied to the object as it moves through a distance x. The angle θ between the force direction and the direction of motion is likewise constant.



The "work" done by the force is defined to be

$$W = Fx \cos \theta$$

where F and x are the *magnitudes* of the force and displacement vectors, i.e., F and x are positive numbers. The cosine term can be positive, negative, or zero, so work W is likewise either positive, negative, or zero.

The equation above assumes that the magnitude and direction of the applied force don't change.

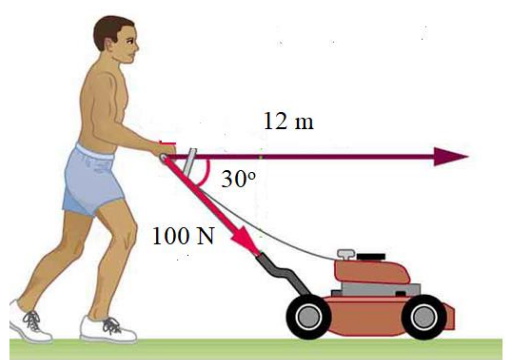
Units of work: newton-meters (N-m)
The SI unit of work is the "joule":

1 "joule" (J) = 1 N-m

Example:

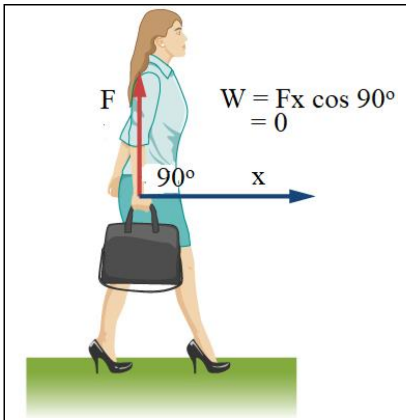
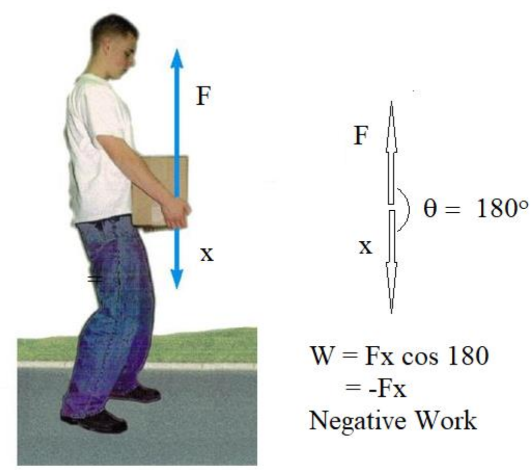
The person in the figure below is exerting a constant force F = 100 N on the lawnmower, always pushing down and to the right at an angle of 30° below the horizontal.

How much work does he do in pushing the lawnmower 12 meters?



$$\begin{aligned} W &= Fx \cos \theta \\ &= (100\text{ N})(12\text{ m}) \cos 30 \\ &= 103.9\text{ N-m} \\ &= 103.9\text{ J} \end{aligned}$$

More Work Examples

 This person *feels like* she's doing work carrying the heavy bag, but based on the physicist's definition of work, she's not doing work.	Lowering Block  Person pulls upward on block as he lowers it to the ground. The angle between the person's lifting force and the direction of motion is 180°. The work he does is negative.

Work Done by Non-Constant Forces

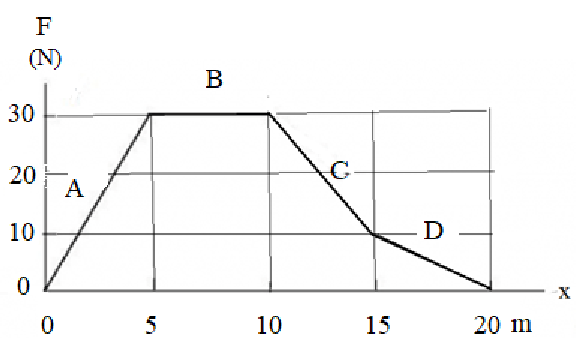
If the force is changing in a constant manner (*linearly*), the work done may be calculated by using the *average* value (the middle value) of the force:

$W = \bar{F}\, x \cos\theta$
$\quad = ½ (F_o + F)\, x \cos\theta$

If the force points in the same direction as the motion, $\theta = 0$, and

$W = \bar{F}\, x \cos 0$
$\quad = \bar{F}\, x$

<u>Example</u>:



Assuming θ = 0, and referring to the force versus displacement graph above, what work is done from x = 0 to x = 20 m?

The average force over each of the four displacement is found using the middle-value rule.

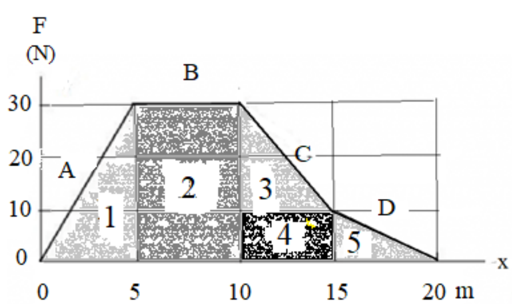
A: ½ (0 + 30) (5) = 75 J
B: 30 (5) = 150 J
C: ½ (30 + 10) (5) = 100 J
D: 5 (5) = 25 J

Total: 350 J

In the example at the left, the work done is visualized as being sum of four separate works, using the middle-value rule to determine the average force.

Another way of obtaining the four works is to use the fact that the *work done is the area under the force-displacement curve*.

The area under the curve above is the area of three triangles and two rectangles.



Three triangles, two rectangles

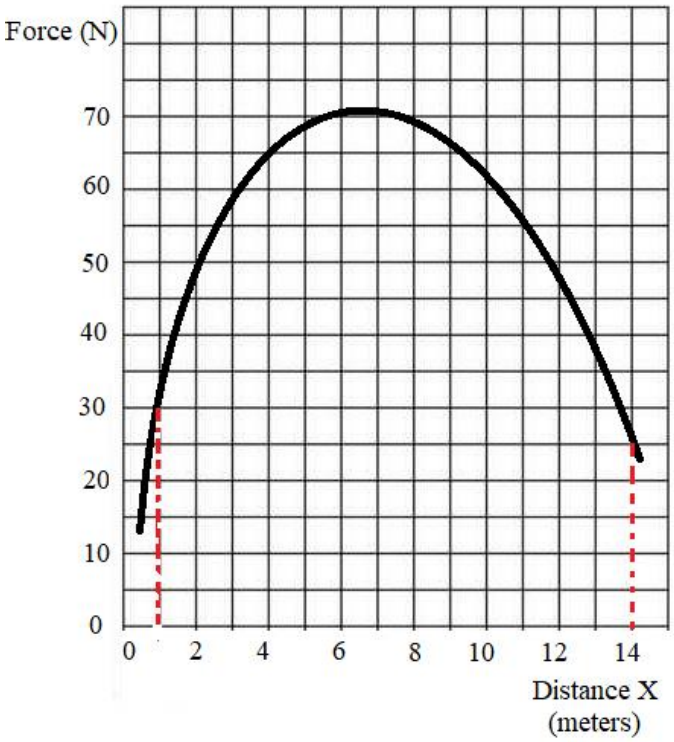
Calculate those five areas to confirm that the total area is 350 J.

1. ½ (5) (30) = 75 J
2. (5) (30) = 150 J
3. ½ (5) (20) = 50 J
4. (5) (10) = 50 J
5. ½ (5) (10) = 25 J

Total Area = 350 J

When regions under the force curve cannot be seen as a set of rectangles and triangles, but the curve's equation is nevertheless known, then calculus can be used to determine the total area. Acquiring the skill to do this is beyond the scope of this course.

Graphical Method of Determining Work



If the force versus distance curve doesn't have an easily discernible equation that calculus can use to "integrate" to obtain the exact area under the curve, there is a way to approximate the area (the work done). See below.

Example:

What is the work done on an object by the force graphed at the left as the object moves from x = 1.0 m to x = 14.0 m?

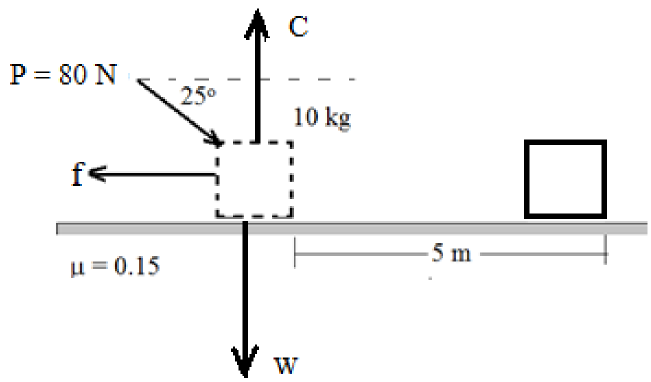
Each grid square on the graph at the left has a width of 1.0 m and a height of 5.0 N, so each square has an area of 5.0 N-m (5.0 J). To find the area, count the number of squares and multiply by 5.0 J to get the work done.

A careful count shows that there are about 140 squares (including fractions of squares), so the estimated work done is 140(5.0) = 700 J.

Using motion and force sensors in lab, and apps that swiftly determine work done is a lot easier than using the grid-square method discussed above.

Example:

Four forces act on the block below, which is sliding to the right. Find the four different works and the total work.



The Four Forces:

Contact Force: **C**
Friction: **f**
Weight: **w**
Push: **P**

The works done by the contact force and weight force are zero because those forces are perpendicular to the displacement vector: cos 90° = 0.

Work done by weight force: zero
Work done by contact force: zero

(Remainder of the solution is provided at the right.)

(continuing example at left)

Calculate Contact Force C

The block's vertical acceleration is zero, so the sum of the vertical components of the forces is zero:

$$C_y + P_y + f_y + w_y = 0$$
$$C - 80 \sin 25 + 0 - 98 = 0$$
$$C = 131.81 \text{ N}$$

....

Calculate Friction

$f = \mu C$
$= 0.15\ (131.81)$
$= 19.77$ N

....

Calculate Work Done by Friction

W= 19.77 (5.0) cos 180
= -98.85 J

....

Work Done by Pushing Force

W = 80 (5.0) cos 25
= 362.52 J

....

Total Work

W = -98.85 J + 362.52 J
= 263.67 J

Average Power

The average power output of a machine, animal, or person during a period of time is the amount of work it does divided by the time required to do the work:

$\bar{P} = W/t$
Units: joule/s (J/s)
1 “watt” = 1 J/s
The “watt” is the SI unit of power.

“Watts” may be abbreviated “W,” provided there could be no confusion with the symbol for work.

In the 18th century the amount of work a horse could do in one second (the horse’s power) was measured. The result is

1.0 “horsepower” (hp) = 746 watts
The “horsepower” is a non-SI unit.

<u>Example</u>:

1000 J of work is done by a machine in 10 seconds. What was its average output power in hp?

$\bar{P}$ = (1000 J)/ (10 s)
= 100 J/s
= 100 watts
= 100 W

100 W / (746 W/hp) = 0.13 hp

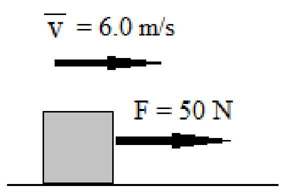
Another Average Power Equation

$$\bar{P} = W/t$$
$$= (Fx \cos\theta)/t$$
$$= F(x/t)\cos\theta$$
$$= F\bar{v}\cos\theta$$
$$\bar{P} = F\bar{v}\cos\theta$$

<u>Example</u>:

Over a period of time a machine applies a constant force of 50 N along the direction of motion ($\theta = 0$). The average speed of the object is 6.0 m/s.

What was the average output power of the machine?



$$\bar{P} = F\bar{v}\cos\theta$$
$$= 50\,(6.0)\cos 0$$
$$= 300 \text{ watts}$$

Power Output of a Human

<u>Example</u>:

Is it possible for a person to “work like a horse”?

Answer:

Yes, but only for a short period of time. World-champion weight-lifters are able to lift 200-kg (440 lbs) weights over their heads in one continuous movement from floor to above their heads in about one second. The distance moved upward is about 2.0 meters.

What is the weight-lifter’s average power output during that one-second period of time?

$F = mg$
$\quad = 200\ (9.8)$
$\quad = 1960\ \text{N}$
$x = 2\ \text{m}$

$\text{Work} = Fx \cos\theta$
$\qquad = 1960\ (2.0) \cos(0^{\circ})$
$\qquad = 3920\ \text{J}$
$\text{Time} = 1.0\ \text{s}$

$\text{Average Power} = \text{Work/Time}$
$\qquad = 3920\ \text{J}/1.0\ \text{s}$
$\qquad = 3920\ \text{J/s}$
$\qquad = 3920\ \text{W}$

$3920\ \text{W} /(746\ \text{W/hp}) = 5.25\ \text{hp}$

The weight-lifter does the work of about five horses over a one-second period of time.

Olympic Gold-Medal runner Osain Bolt’s output power is about 2.8 hp, much less than the weight-lifter’s, but Bolt’s effort was sustained over a period ten times longer.

Instantaneous Power Output

Recall the *average* power output of a constant force acting in the same direction as the direction of motion is

$\bar{P} = F\,\bar{v}$
$\bar{v} = ½\,(v_o + v)$

If the time interval t which the velocity is changing is virtually zero, i.e., the time interval is virtually an "instant" in time, then $v \cong v_o$ and

$\bar{v} \cong ½\,(v + v)$
$\quad \cong v$

Finally,

$\bar{P} = F\,\bar{v}$
$\quad = Fv$

But, the time interval over which $\bar{P}$ is calculated is infinitesimally small, so we may regard $\bar{P}$ as the instantaneous power output, and remove the averaging bar:

$P = Fv$

<u>Example</u>:

What is the instantaneous output power of a machine that's applying a 20-N force to an object at an instant when the object's speed is 2.5 m/s?

$P = Fv$
$\quad = 20\,(2.5)$
$\quad = 50$ watts

Example A:

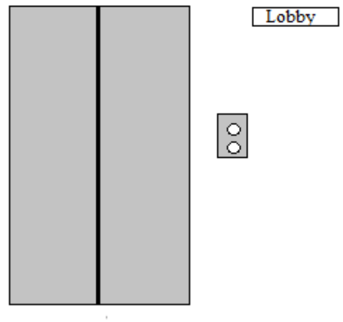
The cable lifting an elevator cab with passengers is exerting an upward force of 10,000 N. At a certain instant the elevator is moving upward at a speed of 2.0 m/s. What is the elevator motor's instantaneous output power, in horsepower, at that instant?

P = Fv
= 10,000 (2.0)
= 20,000 W

Convert to horsepower:

P = 20,000 W /(746 W/hp)
= 26.8 hp

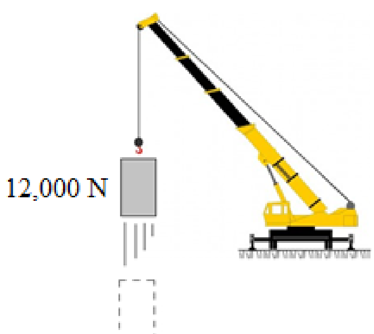
Elevator



Example B:

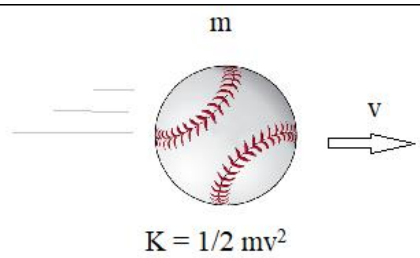
At what constant speed can a crane, operating at 10 horsepower, lift a 12,000-newton object upward?

10 hp (746 W/hp) = 7460 W



12,000 v = 7460 watts
v = 0.62 m/s

Kinetic Energy



v = velocity magnitude (speed)

Kinetic energy is a non-negative, directionless quantity.

Example:

Prove that the units of kinetic energy are the same as the units of work: joules.

Below we use the fact that the units **kg-(m/s^2)** are mass times acceleration units, i.e., they're units of force: newtons (N).

An object of mass m = 5.0 kg is moving at speed 4.0 m/s. What is its kinetic energy?

K = ½ (5.0 kg) (4.0 m/s)^2
= 40.0 kg-m^2/s^2
= 40.0 **(kg-m/s^2)** m (factoring out an "m")
= 40.0 **N**-m
= 40.0 J

Kinetic energy units and work units are the same: *joules*.

Example:

The kinetic energy of an object is 6000 J. Forces are then brought to bear on the object, which result in the object's speed tripling.

By how much does the object's kinetic energy change?

$$K_o = \tfrac{1}{2} mv^2$$
$$= 6000 \text{ J}$$

$$K = \tfrac{1}{2} m (3v)^2$$
$$= 9 (\tfrac{1}{2} mv^2)$$
$$= 9 (6000)$$
$$= 54{,}000 \text{ J}$$

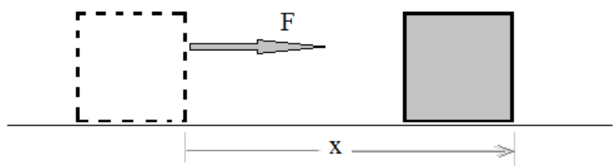
$$\Delta K = K - K_o$$
$$= 54{,}000 - 6{,}000$$
$$= 48{,}000 \text{ J}$$

The Work-Kinetic Energy Theorem

Recall from Chapter 2:

$v^2 = v_o^2 + 2\,ax$
$ax = ½\,(v^2 - v_o^2)$ (1)
(Come back to this later.)

The force below is the total force, so the work it will do will be the total work.



$W = Fx \cos\theta$
$= Fx \cos 0$
$= Fx$
$= (ma)\,x$
$= m\,(ax)$
$= m\,[½\,(v^2 - v_o^2)]$ Using Equation (1)
$= ½\,mv^2 - ½\,mv_o^2$
$= K - K_o$
$= \Delta K$

The Work-Kinetic Energy Theorem in words:

"The *total* work done on an object equals its change in kinetic energy."

Example A:

An object has a kinetic energy of 10.0 J. How much work would have to be done on the object to quadruple its speed?

$$
\begin{aligned}
W &= K - K_o \\
&= \tfrac{1}{2} m (4v)^2 - \tfrac{1}{2} mv^2 \\
&= 16 \left(\tfrac{1}{2} mv^2 \right) - \tfrac{1}{2} mv^2 \\
&= 15 \left(\tfrac{1}{2} mv^2 \right)| \\
&= 15 (10.0 \text{ J}) \\
&= 150 \text{ J}
\end{aligned}
$$

Example B:

An 8-kg object initially has a kinetic energy of 400 J. Six hundred joules of work are done on it.

(a) What is the final kinetic energy?

$$
\begin{aligned}
K - K_o &= W \\
K - 400 &= 600 \\
K &= 1000 \text{ J}
\end{aligned}
$$

(b) What is the final speed?

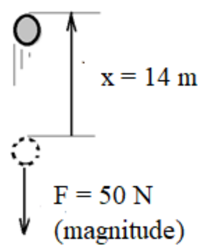
$$
\begin{aligned}
\tfrac{1}{2} (8)v^2 &= 1000 \\
v &= 15.81 \text{ m/s}
\end{aligned}
$$

Example A:

A 6-kg object's speed needs to be changed from 3 m/s to 8 m/s. How much work must be done on it?

$W = K - K_o$
$= \frac{1}{2}(6)8^2 - \frac{1}{2}(6)3^2$
$= 165$ J

Example B:

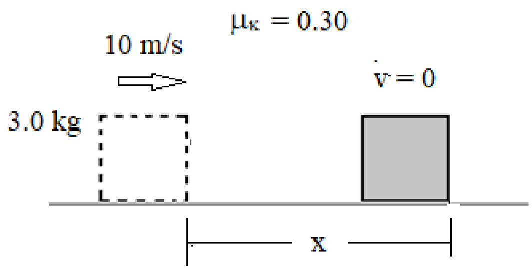


An object weighing 50 N is rising freely; during a period of time its elevation increases by 14 meters.

What was the change in its kinetic energy?

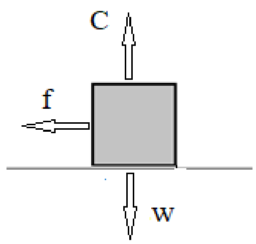
$\Delta K = W$
$= Fx \cos\theta$
$= 50\ x \cos 180$
$= (50)(14) \cos 180$
$= -700$ J

Example:



A 3.0 kg block moving at 10.0 m/s is sliding on a horizontal surface. The coefficient of kinetic friction between the block and the surface is 0.30. Through how many meters will the block travel before coming to rest?

$w = (3.0\text{ kg})(9.8\text{ m/s}^2)$
$= 29.4\text{ N}$



The vertical acceleration is zero, so the sum of the three y-components of forces is zero:

$C_y + f_y + w_y = 0$
$C + 0 - 29.4\text{ N} = 0$
$C = 29.4\text{ N}$

$f = 0.30\,(29.4)$
$= 8.82\text{ N}$

The contact force and the weight force do zero work because $\cos\theta = 0$ in each case; the only force doing work is the frictional force:

$W = 8.82\,(x)\cos 180$
$= -8.82\,x$

Apply the Work-Kinetic Theorem:

$W = K - K_o$
$-8.82\,x = \frac{1}{2}(3.0)(0)^2 - \frac{1}{2}(3.0)(10.0)^2$
$x = 17.01\text{ m}$