# Physics 23 Chapter 2 

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Horizonal and Vertical Motion


Consider an object moving along the horizontal line above. This line is called "the x-axis." Various points along that line mark distances to the right and left from the " 0 " reference zero mark, and are called "x-coordinates." The "units" of these coordinates are "meters" (m). The meter (m) unit is the Standard International (SI) Unit for length and distance, and is approximately 3.28 feet.

## Initial $x$-Coordinate

## $\mathrm{X}_{\mathrm{O}}$

The moving object will be at some initial x-coordinate when our observation of the motion begins, at which time we usually set our clock to zero. The symbol we use to represent the initial x -coordinate is $\mathrm{x}_{\mathrm{o}}$.

The initial $x$-coordinate could be positive, negative, or zero.

As first mentioned in Chapter 1, distances, lengths, are measured in "meters," which are the standard international units of distance, and length. We add to that


The "meter" is the SI unit of distance and length. list x-coordinates, which are likewise measured in meters.

## Final x-Coordinate

X
The moving object will be at a certain later x -coordinate at the end of its travel time. This x coordinate could be positive, negative, and even zero. That later x-coordinate is sometimes called the "final" x-coordinate, and is symbolized as "x," but the word "final" doesn't necessarily imply that the motion ceases; it only means that we are only interested in analyzing the motion up to that point in time.

## Displacement

$\Delta x$

We calculate the "displacement" of an object as shown below:


Average Velocity

The "average velocity" of an object during some time period is defined to be its displacement, divided by the time it took to make that displacement.

$$
\bar{v}=\Delta x / t
$$

Rewritten, this equation becomes

$$
\Delta \mathbf{x}=\overline{\mathrm{v}} \mathbf{t}
$$

## Time Units

As noted in Chapter 1, the standard international units of
 time are seconds ( $s$ ). Clock times are symbolized as "t."

## Example A:

During a five second period an object's displacement is $\Delta x=30 \mathrm{~m}$. What was its average velocity?
$\overline{\mathrm{v}}=\Delta \mathrm{x} / \mathrm{t}$
$=30 \mathrm{~m} / 5 \mathrm{~s}$
$=6 \mathrm{~m} / \mathrm{s}$
Note: the SI units of average velocity are meters per second ( $\mathrm{m} / \mathrm{s}$ ).

## Example B:

During a six-second period of time, an object undergoes a displacement $\Delta x=-180 m$ What was its average velocity?

$$
\begin{aligned}
\overline{\mathrm{v}} & =\Delta \mathrm{x} / \mathrm{t} \\
& =-180 \mathrm{~m} / 6 \mathrm{~s} \\
& =-30 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Average velocities can be negative, as well as positive. They even can be zero.

## Example A:

An object initially moving to the right initially at $\mathrm{x}_{\mathrm{o}}=120 \mathrm{~m}$ slows down and eventually comes to a momentary rest, then reverses direction, heading now to the left. It eventually reaches its initial location at $\mathrm{x}=120 \mathrm{~m}$. The entire trip took eight seconds. What was the object's average velocity?

$$
\begin{aligned}
\overline{\mathrm{v}} & =\Delta \mathrm{x} / \mathrm{t} \\
& =\left(\mathrm{x}-\mathrm{x}_{\mathrm{o}}\right) / \mathrm{t} \\
& =(120 \mathrm{~m}-120 \mathrm{~m}) / 8 \mathrm{~s} \\
& =0 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## Example B:

An object moving to the right travels 36 meter in 12 seconds.

What was its average velocity?
$\overline{\mathrm{v}}=\Delta \mathrm{x} / \mathrm{t}$
Knowledge of its starting and final locations on the x -axis is not require, because $\Delta \mathrm{x}=36 \mathrm{~m}$, no matter what were those coordinates:

$$
\begin{aligned}
\Delta \mathrm{x} & =36 \mathrm{~m} \\
\overline{\mathrm{v}} & =36 \mathrm{~m} / 12 \mathrm{~s} \\
& =3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## Example C:

Over a 10-period of time an object undertakes a two-part journey, traveling to the right initially at $x_{0}=44$ and slowing down to a momentary rest at $x=76 \mathrm{~m}$, then reversing direction, and, finally, returning back to its original location at $\mathrm{x}=44 \mathrm{~m}$.
(a) What was the object's average velocity?
$\Delta \mathrm{x}=\mathrm{x}-\mathrm{x}_{\mathrm{o}}$
$=44 \mathrm{~m}-44 \mathrm{~m}$
$=0$
$\bar{v}=\Delta x / t$
$=0$
(b) What distance did it travel?

32 meters to the right, then 32 meters to the left: total distance $=64$ meters

Note: displacement is not the same as distance traveled when there is a change in the direction of motion.

## Example A:

Over a 4-second time period, an object's average velocity was $5 \mathrm{~m} / \mathrm{s}$.
(a) What was its displacement?

$$
\begin{aligned}
\Delta \mathrm{x} & =\overline{\mathrm{v}} \mathrm{t} \\
& =(5 \mathrm{~m} / \mathrm{s})(4 \mathrm{~s}) \\
& =20 \mathrm{~m}
\end{aligned}
$$

(b) There is not enough information given to calculate the distance traveled. The data given above is consistent with any number of possible reversals of direction.

## Example B:

During a period of time, at the end of which a moving object's displacement during that time was $\Delta x=24 \mathrm{~m}$, its average velocity was $4 \mathrm{~m} / \mathrm{s}$. What was this object's travel time?

$$
\begin{aligned}
\mathrm{t} & =\Delta \mathrm{x} / \overline{\mathrm{v}} \\
& =24 \mathrm{~m} / 4 \mathrm{~m} / \mathrm{s} \\
& =6 \mathrm{~s}
\end{aligned}
$$

## Instantaneous Velocity

| "Instantaneous velocity" is not the same as average velocity. |
| :--- | :--- |
| The "instantaneous" velocity, for example, of an automobile |
| moving along a road is the reading of its speedometer (a |
| positive number) at some instant, prefixed by a negative sign if |
| the car is moving in the negative direction (to the left); |
| otherwise, if it's moving to the right, its velocity is the positive |
| speedometer reading. |

We symbolize an object's initial and final velocities as shown below:

> Initial Velocity: $\mathrm{v}_{\mathrm{o}}$
> Final Velocity:

The zero subscript on the initial velocity symbol above is suggestive of a stopwatch being set to zero when the observation of the motion begins. An example of a moving object's initial and final velocities is found in the figure below:


As illustrated in the figure below, and mentioned above, objects with positive velocities are moving to the right, in the positive-x direction, while objects with negative velocities are moving to the left, in the negative-x direction.

The figure below illustrates the sign rule for velocities: Objects moving to the right have positive velocities, while objects moving to the left have negative velocities.


## Middle-Value Rule

We saw earlier how to calculate the average velocity of an object using the equation below:

$$
\overline{\mathrm{v}}=\Delta \mathrm{x} / \mathrm{t}
$$

There is another way to calculate the average velocity.
The average velocity is found using the "middle-value" rule:

$$
\overline{\mathrm{v}}=1 / 2\left(\mathrm{v}_{0}+\mathrm{v}\right)
$$

This equation gives a number that is mid-way between the initial and final velocities.

| Example A: | Example B: |
| :---: | :---: |
| An object's velocity changes from $20 \mathrm{~m} / \mathrm{s}$ to $80 \mathrm{~m} / \mathrm{s}$. What was its average velocity? | The average velocity of an object is $24 \mathrm{~m} / \mathrm{s}$; its final velocity is $10 \mathrm{~m} / \mathrm{s}$. |
| $\begin{aligned} \overline{\mathrm{v}} & =1 / 2(20+80) \\ & =50 \mathrm{~m} / \mathrm{s} \end{aligned}$ | What was its initial velocity? $\overline{\mathrm{v}}=1 / 2\left(\mathrm{v}_{\mathrm{o}}+\mathrm{v}\right)$ |
| Note that 50 is "midway" between 20 and 80 : It's 30 above 20, and 30 below 80 . | $\begin{aligned} & 24=1 / 2\left(\mathrm{v}_{\mathrm{o}}+10\right) \\ & \mathrm{v}_{\mathrm{o}}=38 \mathrm{~m} / \mathrm{s} \end{aligned}$ |
| Example C: | Example D: |
| An object is initially at $x_{0}=8 \mathrm{~m}$. During the next four seconds, its velocity changes from $-16 \mathrm{~m} / \mathrm{s}$ to $10 \mathrm{~m} / \mathrm{s}$. Where is the object at the end of this period? | Over a certain travel time the velocity of an object changes from $4.0 \mathrm{~m} / \mathrm{s}$ to $26.0 \mathrm{~m} / \mathrm{s}$. During this period, the object's displacement was 75 meters. |
| $\Delta \mathrm{x}=\overline{\mathrm{v}}_{\mathrm{t}}$ | What was the travel time? |
| $x=-4 m$ | $\begin{aligned} 75 & =1 / 2(4.0+26.0) \mathrm{t} \\ \mathrm{t} & =5.0 \text { seconds } \end{aligned}$ |

## Acceleration

When an object is accelerating horizontally, its velocity is changing-either increasing, or decreasing. The average acceleration is the change in velocity, divided by the time required for the change to occur:

$$
\overline{\mathrm{a}}=\left(\mathrm{v}-\mathrm{v}_{\mathrm{o}}\right) / \mathrm{t}
$$

The SI units of acceleration are meters per second, per second, or $\mathrm{m} / \mathrm{s}^{2}$.
For the remainder of this course, our accelerations will not be changing, so it will make no sense to talk about the average value of a thing whose value never changes. Therefore, we can remove the "bar" from above the "a":

$$
\mathrm{a}=\left(\mathrm{v}-\mathrm{v}_{\mathrm{o}}\right) / \mathrm{t}
$$

The velocity equation below is the same as the one above, re-arranged.

$$
\mathbf{v}=\mathbf{v}_{\mathbf{o}}+\mathbf{a t}
$$

The tables below illustrate accelerations that are positive, negative, and zero.

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{o}}=6 \mathrm{~m} / \mathrm{s} \\
& \mathrm{v}_{\mathrm{o}}=15 \mathrm{~m} / \mathrm{s} \\
& \mathrm{a}=-3 \mathrm{~m} / \mathrm{s}^{2} \\
& \mathrm{v}_{\mathrm{o}}=15 \mathrm{~m} / \mathrm{s} \\
& \mathrm{a}=3 \mathrm{~m} / \mathrm{s}^{2} \\
& \mathrm{v}=\mathrm{v}_{\mathrm{o}}+\text { at } \\
& =6+3 \mathrm{t} \\
& \mathrm{a}=0 \\
& \mathrm{v}=\mathrm{v}_{\mathrm{o}}+\mathrm{at} \\
& =15+0
\end{aligned}
$$

Table 1

Table 2

| t <br> $(\mathrm{s})$ | v <br> $(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: |
| 0 | 15 |
| 1 | 12 |
| 2 | 9 |
| 3 | 6 |
| 4 | 3 |
| 5 | 0 |
| 6 | -3 |
| 7 | -6 |


| t <br> $(\mathrm{s})$ | v <br> $(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: |
| 0 | 6 |
| 1 | 9 |
| 2 | 12 |
| 3 | 15 |
| 4 | 18 |
| 5 | 21 |
| 6 | 24 |
| 7 | 27 |


| t <br> $(\mathrm{s})$ | v <br> $(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: |
| 0 | 15 |
| 1 | 15 |
| 2 | 15 |
| 3 | 15 |
| 4 | 15 |
| 5 | 15 |
| 6 | 15 |
| 7 | 15 |

In Table 2, note that until time $\mathrm{t}=5$ seconds is reached, the object's velocity is positive, indicating that it's moving to the right. After 5 seconds, the velocity is negative, meaning that the object reversed direction and now is moving to the left.

## Distance vs Displacement

The distance traveled by an object is sometimes the absolute value of its displacement, and sometimes, not. Only if there is no change in direction will the distance traveled equal the absolute value of the displacement. The examples below will illustrate this.

| Example A: | Example B: |
| :---: | :---: |
| An object travels to the right from | An object undergoes a two-part journey, traveling to the right from its initial location at point A , arriving at point $B$, then reversing direction of travel and ending up at its final location at point $\mathrm{C}, 7$ meters to the right of its starting point at A |
| +8 m . The distance it travels is likewise 8 m . |  |
| likewise 8 m . | A 12 m |
| Example C: | The object ends up seven meters farther to the right of its initial location, so its displacement during this |
| An object travels to the left from point A to point $B$. Its displacement is -9 m . The distance it travels is $\|-9 \mathrm{~m}\|=9 \mathrm{~m}$. | journey is +7 m . However, the distance traveled is $12+5=17 \mathrm{~m}$. |
|  | $12+5=17 \mathrm{~m} .$ |
| $\mathrm{B} \cdot<\Longleftarrow \cdot \frac{9 \mathrm{~m}}{\mathrm{~A}}$ | Clearly--unlike in the two examples at the left--in this case the distance traveled is not the same as the object's displacement. Only if there is no change in direction of movement is the absolute value of the displacement equal to the distance traveled. |


| Example A: | Example B: |
| :---: | :---: |
| An object's initial velocity is $30 \mathrm{~m} / \mathrm{s}$, and is accelerating at $2 \mathrm{~m} / \mathrm{s}^{2}$. | An object's velocity is initially $25 \mathrm{~m} / \mathrm{s}$. It then begins accelerating at the rate of $5 \mathrm{~m} / \mathrm{s}^{2}$. |
| After how many seconds will the velocity be $60 \mathrm{~m} / \mathrm{s}$ ? | (a) What is its velocity nine seconds later? $\begin{aligned} \mathrm{v} & =\mathrm{v}_{\mathrm{o}}+\mathrm{at} \\ & =25 \mathrm{~m} / \mathrm{s}+\left(5 \mathrm{~m} / \mathrm{s}^{2}\right) 9 \mathrm{~s} \\ & =70 \mathrm{~m} / \mathrm{s} \end{aligned}$ |
| $\mathrm{t}=15 \mathrm{~s}$ | (b) How far did the object travel in that time? |
|  | $\begin{aligned} \Delta \mathrm{x} & =\overline{\mathrm{v}} \mathrm{t} \\ & =1 / 2(25+70) 9 \\ & =427.5 \mathrm{~m} \end{aligned}$ |
|  | If an object is moving only to the right during a period time, the distance it travels therefore is the same as its displacement. Thus, the distance traveled in this case is 405 m . |


| Example C: | Example D: |
| :---: | :---: |
| An object initially at $x_{0}=20 \mathrm{~m}$ has a velocity $\mathrm{v}_{\mathrm{o}}=6 \mathrm{~m} / \mathrm{s}$. Three seconds later its velocity is $\mathrm{v}=18 \mathrm{~m} / \mathrm{s}$. | An object initially traveling at a velocity of $40 \mathrm{~m} / \mathrm{s}$ begins to slow down at the rate $-4 \mathrm{~m} / \mathrm{s}^{2}$. How far will the object have traveled when it comes to rest? |
| What is its x -coordinate at that time? | At rest, the object's velocity is zero: |
| $\begin{aligned} \overline{\mathrm{v}} & =1 / 2\left(\mathrm{v}_{\mathrm{o}}+\mathrm{v}\right) \\ & =1 / 2(6+18)\end{aligned}$ | $0=40-4 \mathrm{t}$ |
|  | $\mathrm{t}=10 \mathrm{~s}$ |
| $\begin{aligned} \Delta \mathrm{x} & =\overline{\mathrm{v}} \mathrm{t} \\ & =(12 \mathrm{~m} / \mathrm{s})(3 \mathrm{~s}) \\ & =36 \mathrm{~m}\end{aligned}$ | $\overline{\mathrm{v}}=1 / 2\left(\mathrm{v}_{\mathrm{o}}+\mathrm{v}\right)$ |
|  | $=1 / 2(40+0)$ |
|  | $=20 \mathrm{~m} / \mathrm{s}$ |
| $=56 \mathrm{~m}$ | $\Delta \mathrm{x}=\overline{\mathrm{v}} \mathrm{t}$ |
|  | $\begin{aligned} & =(20 \mathrm{~m} / \mathrm{s})(10 \mathrm{~s}) \\ & =200 \mathrm{~m} \end{aligned}$ |
|  | The object is moving only to the right as it comes to rest, so the distance traveled equals its displacement: 200 m . |

Below is derived one more motion equation.

First, recall the two equations below:

$$
\begin{align*}
\Delta \mathrm{x} & =1 / 2\left(\mathrm{v}_{\mathrm{o}}+\mathrm{v}\right) \mathrm{t}  \tag{1}\\
\mathrm{v} & =\mathrm{v}_{\mathrm{o}}+\mathrm{at} \tag{2}
\end{align*}
$$

Use (2) to replace v in (1):

$$
\begin{aligned}
\Delta x & =1 / 2\left[v_{0}+\left(v_{0}+a t\right)\right] t \\
& =1 / 2\left(2 v_{0}+a t\right) t \\
& =v_{0} t+1 / 2 a t^{2} \\
x-x_{0} & =v_{0} t+1 / 2 a t^{2} \\
\mathbf{x}= & \mathbf{x}_{\mathbf{0}}+v_{\mathbf{o}} \mathbf{t}+1 / 2 \mathbf{a t}^{2}
\end{aligned}
$$

## Example:

An object initially at $x_{0}=30 \mathrm{~m}$ is moving at $6 \mathrm{~m} / \mathrm{s}$ and accelerating at 3 $\mathrm{m} / \mathrm{s}^{2}$. Where is the object 8 seconds later?

$$
\begin{aligned}
\mathrm{x} & =\mathrm{x}_{\mathrm{o}}+\mathrm{v}_{\mathrm{o}} \mathrm{t}+1 / 2 \mathrm{at}^{2} \\
& =30+6(8)+1 / 2(3) 8^{2} \\
& =174 \mathrm{~m}
\end{aligned}
$$

## Example:

An object at $x_{o}=30 \mathrm{~m}$ is initially moving at $6 \mathrm{~m} / \mathrm{s}$ velocity. The object then begins accelerating at the rate of $4.0 \mathrm{~m} / \mathrm{s}^{2}$.

After how many seconds will the object be at $\mathrm{x}=160 \mathrm{~m}$ ?

$$
\begin{aligned}
\mathrm{x} & =\mathrm{x}_{\mathrm{o}}+\mathrm{v}_{\mathrm{o}} \mathrm{t}+1 / 2 \mathrm{at}^{2} \\
160 & =30+6 \mathrm{t}+1 / 2(4) \mathrm{t}^{2} \\
2 \mathrm{t}^{2} & +6 \mathrm{t}-130=0
\end{aligned}
$$

This is a quadratic equation, so it has two solutions:
Using a calculator equation solver, we get $\mathrm{t}=6.70 \mathrm{~s}$ and $\mathrm{t}=-9.70 \mathrm{~s}$.

We ignore the negative solution as non-physical.

The equation is alternatively solved in Example B using the cumbersome quadratic formula.

## Example A: <br> $2 \mathrm{t}^{2}+6 \mathrm{t}-130=0$

Recall the general form of the quadratic equation, and its two solutions:

If $a x^{2}+b x+c=0$
$x=\left[-b \pm\left(b^{2}-4 a c\right)^{1 / 2}\right] / 2 a$
Solve the equation below:
$2 \mathrm{t}^{2}+6 \mathrm{t}-130=0$
$\mathrm{a}=2$
$b=6$
$\mathrm{c}=-130$
$\mathrm{x}=\left[-6+(36+1040)^{1 / 2}\right] / 4 \quad$ (Choosing the positive sign.)
$=6.70 \mathrm{~s}$
$x=\left[-6-(36+1040)^{1 / 2}\right] / 4 \quad$ (Choosing the negative sign.)
$=-9.70 \mathrm{~s}$ (ignored)

## Example B:

A moving object begins to accelerate at $1.2 \mathrm{~m} / \mathrm{s}^{2}$. After 4.0 seconds the object has traveled 20 m . What was its initial velocity?

$$
\begin{aligned}
& \Delta \mathrm{x}=\mathrm{v}_{\mathrm{o}} \mathrm{t}+1 / 2 \mathrm{at}^{2} \\
& 20=\mathrm{v}_{\mathrm{o}}(4.0)+1 / 2(1.2)(4.0)^{2} \\
& \mathrm{v}_{\mathrm{o}}=2.6 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## Example:

A runner traveling initially at $3.0 \mathrm{~m} / \mathrm{s}$ is accelerating at $0.10 \mathrm{~m} / \mathrm{s}^{2}$. After how many seconds will she have run 100 meters?
$100=3.0 \mathrm{t}+1 / 2(0.10) \mathrm{t}^{2}$
Use the quadratic formula to solve for $t$, or the equation solving tool on your calculator.
$\mathrm{t}=23.85 \mathrm{~s}$ and -83.85 s
We rule out negative travel times as impossible, so we're left with $t=23.85 \mathrm{~s}$ as the answer.

## Horizontal Motion Equation Summary

$$
\begin{array}{|l}
\overline{\mathrm{v}}=1 / 2\left(\mathrm{v}_{\mathrm{o}}+\mathrm{v}\right) \\
\hline \Delta \mathrm{x}=\mathrm{x}-\mathrm{x}_{\mathrm{o}} \\
\hline \Delta \mathrm{x}=\overline{\mathrm{v}} \mathrm{t} \\
\hline \Delta \mathrm{x}=1 / 2\left(\mathrm{v}_{\mathrm{o}}+\mathrm{v}\right) \mathrm{t} \\
\hline \mathrm{v}=\mathrm{v}_{\mathrm{o}}+\mathrm{at} \\
\hline \mathrm{x}=\mathrm{x}_{\mathrm{o}}+\mathrm{v}_{\mathrm{o}} \mathrm{t}+1 / 2 \mathrm{at}^{2}
\end{array}
$$

| Example A: | Example B: |
| :---: | :---: |
| An object initially traveling at a velocity of $40 \mathrm{~m} / \mathrm{s}$ begins to accelerate at the rate $a=-4 \mathrm{~m} / \mathrm{s}^{2}$. | In four seconds an object's velocity changes from $12 \mathrm{~m} / \mathrm{s}$ to $36 \mathrm{~m} / \mathrm{s}$. How far will it travel in the next five seconds? |
| How far will it have traveled by the time it comes to momentary rest? | First Four Seconds: $\begin{aligned} \mathrm{v} & =\mathrm{v}_{\mathrm{o}}+\mathrm{at} \\ 36 & =12+\mathrm{a}(4) \end{aligned}$ |
| Losing $4 \mathrm{~m} / \mathrm{s}$ each second, starting with $40 \mathrm{~m} / \mathrm{s}$, would require ten seconds to reduce the velocity to zero: | $\mathrm{a}=6 \mathrm{~m} / \mathrm{s}^{2}$ |
|  | Next Five Seconds: |
| $\begin{aligned} x & =\bar{v} t \\ & =1 / 2\left(v_{0}+v\right) t \end{aligned}$ | The initial velocity for the new time period-- the five-second time period-- is |
| $\begin{aligned} &=200 \mathrm{~m} \\ & \mathrm{~d}=200 \mathrm{~m} \end{aligned}$ | $\mathrm{v}_{\mathrm{o}}=36 \mathrm{~m} / \mathrm{s}$ |
|  | The new final velocity is $\begin{aligned} \mathrm{v} & =\mathrm{v}_{\mathrm{o}}+\mathrm{at} \\ & =36+6(5) \\ \mathrm{v} & =66 \mathrm{~m} / \mathrm{s} \end{aligned}$ |
|  | $\mathrm{x}=\mathrm{v} \mathrm{t}$ |
|  | $\begin{aligned} & =1 / 2\left(\mathrm{v}_{\mathrm{o}}+\mathrm{v}\right) \mathrm{t} \\ & =1 / 2(36+66) 5 \end{aligned}$ |
|  | $=255 \mathrm{~m}$ |
|  | $\mathrm{d}=255 \mathrm{~m}$ |


| $\%$ Video Lecture 1: | Introduction to Horizontal Motion Part One |
| :--- | :--- |
| Video Lecture 2: | Introduction to Horizontal Motion Part Two |

## Problems Involving Two Moving Objects

## Example:

A car is traveling eastward (in the positive x direction) the wrong way on a one-way street at velocity of $20 \mathrm{~m} / \mathrm{s}$ and accelerating at $3 \mathrm{~m} / \mathrm{s}^{2}$.

A bus on the same street 500 m from the car is moving toward the car at $-30 \mathrm{~m} / \mathrm{s}$.


After how many seconds will the two vehicles collide?
Solution:
Let the car's initial location be $\mathrm{x}_{\mathrm{o}}=0$, and the bus's initial location be $\mathrm{x}_{0}=500 \mathrm{~m}$.

Car:
$x=x_{0}+v_{0} t+1 / 2 a t^{2}$
$x=0+20 t+1 / 2(3) t^{2}$
Bus:
$x=x_{0}+v_{0} t+1 / 2 a t^{2}$
$x=500-30 t+0$

The vehicles collide when the x-coordinates of the two vehicles are the same:

$$
\begin{aligned}
& 20 t+1 / 2(3) t^{2}=500-30 t \\
& t=8.05 s
\end{aligned}
$$

## Modifying the Motion Equations

Except for problems involving two moving objects, where each of the objects were initially at different x -coordinates $\mathrm{x}_{\mathrm{o}}$, it will be convenient in going forward to let the initial x -coordinates $\mathrm{x}_{\mathrm{o}}$ be zero. We therefore modify the list of motion equations encountered so far as indicated in the table below:

$$
\mathrm{x}_{\mathrm{o}}=0
$$

| $\bar{v}=1 / 2\left(v_{o}+v\right)$ |
| :--- |
| $x=\bar{v} t$ |
| $x=1 / 2\left(v_{0}+v\right) t$ |
| $v=v_{o}+a t$ |
| $x=v_{0} t+1 / 2 a t^{2}$ |

## One More Motion Equation

$$
\begin{aligned}
v=v_{0}+a t \leadsto t=\left(v-v_{0}\right) / a
\end{aligned} \quad \begin{array}{r}
x=1 / 2\left(v_{0}+v\right)\left(v-v_{0}\right) / a \\
=1 / 2\left(v^{2}-v_{0}{ }^{2}\right) / a
\end{array}
$$

Nowhere in this equation does the time $t$ variable appear, so this is the equation you might use if time $t$ is not given and not asked for.

## Complete List of Horizontal Motion Equations

\[

\]

## Example

An object initially traveling at a velocity of $40 \mathrm{~m} / \mathrm{s}$ begins to accelerate at the rate $\mathrm{a}=-4 \mathrm{~m} / \mathrm{s}^{2}$.

How far will it have traveled by the time it comes to rest?
This problem was solved earlier on Page 9, Example D. We solve it again, here, using a different method.

When the object comes to rest, $\mathrm{v}=0$ :

$$
\begin{aligned}
\mathrm{v}^{2} & =\mathrm{v}_{\mathrm{o}}{ }^{2}+2 \mathrm{ax} \\
0 & =40^{2}+2(-4) \mathrm{x} \\
\mathrm{x} & =200 \mathrm{~m}
\end{aligned}
$$

## Example A:

An object whose initial velocity is $6 \mathrm{~m} / \mathrm{s}$ is accelerating at the rate of $3 \mathrm{~m} / \mathrm{s}^{2}$. Later, the velocity is $30 \mathrm{~m} / \mathrm{s}$.

What is the object's x-coordinate at that moment?

$$
\begin{aligned}
\mathrm{v}^{2} & =\mathrm{v}_{\mathrm{o}}{ }^{2}+2 \mathrm{ax} \\
30^{2} & =6^{2}+2(3) \mathrm{x} \\
\mathrm{x} & =144 \mathrm{~m}
\end{aligned}
$$

## Example B:

In traveling 800 m , a car's velocity increases from $20 \mathrm{~m} / \mathrm{s}$ to $30 \mathrm{~m} / \mathrm{s}$. What was its acceleration?

$$
\begin{aligned}
v^{2} & =v_{o}{ }^{2}+2 a x \\
30^{2} & =20^{2}+2 \mathrm{a}(800) \\
\mathrm{a} & =0.31 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

## Unit Conversion Example

## Example:

Convert the velocity $30 \mathrm{~m} / \mathrm{s}$ to $\mathrm{mi} / \mathrm{hr}(\mathrm{mph})$ units.
It is not necessary to memorize the numbers below. If these values are needed on an exam, they will be provided to the student.
$1 \mathrm{~m}=3.28 \mathrm{ft}$
$1 \mathrm{mi}=5280 \mathrm{ft}$
$1 \mathrm{hr}=3600 \mathrm{~s}$
$(30 \mathrm{~m} / \mathrm{s})(3.28 \mathrm{ft} / \mathrm{m})(3600 \mathrm{~s} / \mathrm{hr}) /(5280 \mathrm{ft} / \mathrm{mi})=67 \mathrm{mi} / \mathrm{hr}$

## Vertical Motion



In this section we study objects that are thrown or fired straight upward or downward, or dropped. The vertical line along which they travel is the $y$-axis.

We will only discuss problems involving single objects, so the zero reference mark will always be wherever the object was when observation of the motion began:
$y_{o}=0$
Objects that are above the reference mark
 on the $y$-axis have positive $y$-coordinates, while objects below the reference mark have negative $y$-coordinates.

Objects moving upward have positive velocities, while objects moving downward have negative velocities.

## Acceleration Due to Gravity

All objects moving upward or downward under only the influence of Earth's pull have an acceleration
$\mathrm{a}=-9.8 \mathrm{~m} / \mathrm{s}^{2}$,
called "the acceleration due to gravity." Such objects are said to be moving "freely," due only to the pull by Earth.

The value, $9.8 \mathrm{~m} / \mathrm{s}^{2}$, occurs so often in problems involving objects rising or falling, that it is often convenient to use the symbol " g " instead of writing $9.8 \mathrm{~m} / \mathrm{s}^{2}$.

Thus, we let
$\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}$
And therefore, for vertical motion,
$a=-g$.

## Illustrating Acceleration due to Gravity

Dropped

| t <br> $(\mathrm{s})$ | v <br> $(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: |
| 0 | 0 |
| 1 | -9.8 |
| 2 | -19.6 |
| 3 | -29.4 |

Thrown Downward

| t |
| :---: | :---: |
| $(\mathrm{s})$ | | v |
| :---: |
| $(\mathrm{m} / \mathrm{s})$ |
| 0 |

Thrown Upward

| $\begin{gathered} \mathrm{t} \\ (\mathrm{~s}) \\ \hline \end{gathered}$ | $\begin{aligned} & \mathrm{v} \\ & (\mathrm{~m} / \mathrm{s}) \end{aligned}$ |
| :---: | :---: |
| 0 | 30.0 |
| 1 | 20.2 |
| 2 | 10.4 |
| 3 | 0.6 |

## Vertical Motion Equations

Recall the horizontal motion equations for single-object motion problems.

| $x=1 / 2\left(v_{0}+v\right) t$ |
| :--- |
| $v=v_{0}+a t$ |
| $x=v_{o} t+1 / 2 a t^{2}$ |
| $v^{2}=v_{0}^{2}+2 a x$ |

The vertical motion equations look the same, except " $y$ " replaces " $x$ ", and the acceleration " $a$ " is replaced by" -g":

$$
\begin{aligned}
& \mathrm{y}=1 / 2\left(\mathrm{v}_{\mathrm{o}}+\mathrm{v}\right) \mathrm{t} \\
& \hline \mathrm{v}=\mathrm{v}_{\mathrm{o}}-\mathrm{gt} \\
& \hline \mathrm{y}=\mathrm{v}_{\mathrm{o}} \mathrm{t}-1 / 2 \mathrm{gt} t^{2} \\
& \mathrm{v}^{2}=\mathrm{v}_{\mathrm{o}}{ }^{2}-2 \mathrm{gy} \\
& \hline
\end{aligned}
$$

## Example:

An object is dropped from the top of a cliff 35 meters high.
The object's motion begins at the top of the building, at $y=0$, and travels to the ground, whose $y$-coordinate is $y=-35 \mathrm{~m}$.
(a) How long does it take to strike the ground?
$y=v_{0} t-1 / 2(9.8) t^{2}$
$-35=0-1 / 2(9.8) t^{2}$
$\mathrm{t}=2.7 \mathrm{~s}$
(b) What will be the object's velocity when it strikes the ground?
$\mathrm{v}=\mathrm{v}_{\mathrm{o}}-\mathrm{gt}$
$=0-9.8$ (2.7)
$=-26.5 \mathrm{~m} / \mathrm{s}$

## Example A:

An object is thrown downward with velocity $\mathrm{v}_{\mathrm{o}}=-16 \mathrm{~m} / \mathrm{s}$ from the top of a building 30 meters high.
(a) How long will it take to strike the ground?
$\mathrm{v}_{\mathrm{o}}=-16 \mathrm{~m} / \mathrm{s}$
$\mathrm{y}=\mathrm{v}_{0} \mathrm{t}-1 / 2 \mathrm{gt}^{2}$
$-30=-16 \mathrm{t}-1 / 2(9.8) \mathrm{t}^{2}$
$\mathrm{t}=1.33 \mathrm{~s}$
(b) How long will it take to strike the ground if it's thrown upward at $\mathrm{v}_{\mathrm{o}}=16 \mathrm{~m} / \mathrm{s}$ ?

$$
\begin{aligned}
-30 & =16 \mathrm{t}-1 / 2(9.8) \mathrm{t}^{2} \\
\mathrm{t} & =4.60 \mathrm{~s}
\end{aligned}
$$

## Example B:

A bullet is fired from a pistol upward at $100 \mathrm{~m} / \mathrm{s}$. What will be the bullet's velocity when it is 30 meters above the pistol?

There are two answers:
$\mathrm{v}^{2}=100^{2}+2(-9.8) 30$
$\mathrm{v}= \pm 97 \mathrm{~m} / \mathrm{s}$
On the way up, $v=97 \mathrm{~m} / \mathrm{s}$.
On the way back down, $v=-97 \mathrm{~m} / \mathrm{s}$.

## Rise Time versus Fall Time

## Example:

An arrow is fired at velocity $\mathrm{v}_{\mathrm{o}}=19.6 \mathrm{~m} / \mathrm{s}$ upward.
(a) What is the "rise time" of the arrow?

The rising period ends when the arrow's upward velocity v is reduced to zero.
$\mathrm{v}=\mathrm{v}_{\mathrm{o}}-\mathrm{gt}$
$0=v_{0}-g t$
$\mathrm{t}=\mathrm{v}_{\mathrm{o}} / \mathrm{g}$
$=(19.6 \mathrm{~m} / \mathrm{s}) / 9.8 \mathrm{~m} / \mathrm{s}^{2}$
$=2.0 \mathrm{~s}$
(b) What is the arrow's total time in the air, i.e., the time it takes the arrow to rise upward, then fall down to the same place (at $\mathrm{y}=0$ ) from which it was fired?
$y=v_{0} t-1 / 2 \operatorname{gt}^{2}$
$0=\mathrm{v}_{\mathrm{o}} \mathrm{t}-1 / 2 \mathrm{gt}^{2}$
$\mathrm{t}=2\left(\mathrm{v}_{\mathrm{o}} / \mathrm{g}\right)$
$=2(19.6) / 9.8$
$=4.0 \mathrm{~s}$
(c) How much time did it take the arrow to fall from its maximum height down to ground?

Fall time $=$ Total Time - Rise Time

$$
\begin{aligned}
& =4.0 \mathrm{~s}-2.0 \mathrm{~s} \\
& =2.0 \mathrm{~s}
\end{aligned}
$$

Note: The fall time equals the rise time.
In the next chapter we will study projectile motion, where we will learn that the fall time of an arcing projectile traveling over level ground also equals its rise time.

