Physics 23 Chapter 14
Ideal Gases

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Avogadro's Number: $N_A = 6.02 \times 10^{23}$ 6.02×10^{23} atoms of any *atomic* substance (such as carbon, silicon, copper, silver, gold) is called a "mole." Likewise, 6.02×10^{23} molecules of a *molecular* substance, such as carbon dioxide, CO_2, methane, CH_4, and water, H_2O is called a "mole."	 Amadeus Avogadro 1776-1856

Example A: 2.5×10^{24} atoms of copper are how many moles? Answer: Let n = number of moles Let N = number of atoms $n = N/N_A$ $= 2.5 \times 10^{24}$ atoms $/6.02 \times 10^{23}$ atoms/mole $= 4.15$ moles	Example B: How many atoms are in 12.5 moles of carbon? Answer: $N = nN_A$ $= (12.5 \text{ moles}) (6.02 \times 10^{23}$ atoms/mole $= 7.525 \times 10^{24}$ atoms

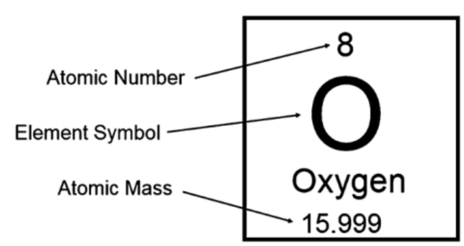
The answers above would apply to *any* element.

Atomic Masses of Elemental Substances

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H 1.008 hydrogen																	2 He 4.003 helium
3 Li 6.94 lithium	4 Be 9.012 beryllium											5 B 10.81 boron	6 C 12.01 carbon	7 N 14.01 nitrogen	8 O 16.00 oxygen	9 F 19.00 fluorine	10 Ne 20.18 neon
11 Na 22.99 sodium	12 Mg 24.31 magnesium											13 Al 26.98 aluminum	14 Si 28.09 silicon	15 P 30.97 phosphorus	16 S 32.06 sulfur	17 Cl 35.45 chlorine	18 Ar 39.95 argon
19 K 39.10 potassium	20 Ca 40.08 calcium	21 Sc 44.96 scandium	22 Ti 47.87 titanium	23 V 50.94 vanadium	24 Cr 52.00 chromium	25 Mn 54.94 manganese	26 Fe 55.85 iron	27 Co 58.93 cobalt	28 Ni 58.69 nickel	29 Cu 63.55 copper	30 Zn 65.38 zinc	31 Ga 69.72 gallium	32 Ge 72.63 germanium	33 As 74.92 arsenic	34 Se 78.97 selenium	35 Br 79.90 bromine	36 Kr 83.80 krypton
37 Rb 85.47 rubidium	38 Sr 87.62 strontium	39 Y 88.91 yttrium	40 Zr 91.22 zirconium	41 Nb 92.91 niobium	42 Mo 95.95 molybdenum	43 Tc [97] technetium	44 Ru 101.1 ruthenium	45 Rh 102.9 rhodium	46 Pd 106.4 palladium	47 Ag 107.9 silver	48 Cd 112.4 cadmium	49 In 114.8 indium	50 Sn 118.7 tin	51 Sb 121.8 antimony	52 Te 127.6 tellurium	53 I 126.9 iodine	54 Xe 131.3 xenon
55 Cs 132.9 cesium	56 Ba 137.3 barium	57-71 La–Lu *	72 Hf 178.5 hafnium	73 Ta 180.9 tantalum	74 W 183.8 tungsten	75 Re 186.2 rhenium	76 Os 190.2 osmium	77 Ir 192.2 iridium	78 Pt 195.1 platinum	79 Au 197.0 gold	80 Hg 200.6 mercury	81 Tl 204.4 thallium	82 Pb 207.2 lead	83 Bi 209.0 bismuth	84 Po [209] polonium	85 At [210] astatine	86 Rn [222] radon

The chart above is not a complete listing of the elements. Useful rule: elements lying in the same column have similar electrical and chemical properties. Column 11, for example: copper, silver, gold.

Moles and Atomic Mass

One mole of an element—in addition to being Avogadro's Number of atoms of an elemental substance, is also a certain number of grams of the element. That particular number is described below: The decimal numbers inside the element boxes above are the "atomic masses" of the element, measured in grams. That number—the atomic mass—represents the number of grams of that element that contain 6.02×10^{23} atoms, i.e., one mole of that element. For example, 15.999 grams of oxygen (O) equal one mole of oxygen. The atomic mass is symbolized as A. "Atomic mass" is sometimes called, "Atomic Weight."	

Example A :

What is the mass of an atom of uranium? A = 238.029 grams.

$$\frac{238.029\ g/\text{mole}}{6.02 \times 10^{23}\ \text{atoms}/\text{mole}} = 3.95 \times 10^{-22}\ g/\text{atom}$$

Example B:

How many atoms are there in 100 grams of copper? (A = 63.55)

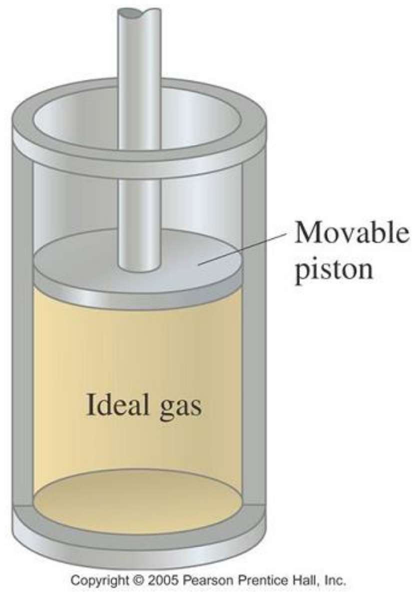
n =100 g / 63.55 g/mole
= 1.57 moles

$1.57\ (6.02 \times 10^{23}) = 9.45 \times 10^{23}$ atoms

The Ideal Gas Law

In this section we will discuss “ideal” monatomic gases, which are hypothetical gases consisting of atoms which neither collide nor bond with each other. Ideal gases don’t exist, but many gases behave as if they were nearly ideal.

In this section, the ideal monatomic gases we will study are typically inside a cylinder with piston (lid), usually “weightless” that is often movable, but sometimes not. Such a cylinder is shown at the right.



The significance of a piston being movable is found in the fact that the piston will rise or fall until the pressure inside matches the external pressure, which is often just atmospheric pressure.

Properties that are relevant to our study of ideal gases in a container are pressure (P), volume (V), the number of atoms (N), the number of moles (n) and the Kelvin temperature (T).

Note: No piston is truly weightless. For our purposes, the word “weightless” means that the piston’s weight is ignorably small compared to other forces in the system.

Other note: Some gas containers have a vent that allows gas to enter or leave. In unvented containers the number of moles doesn’t change.

Ideal Gas Pressure, Temperature and Volume

Relevant Constants, Symbols, Equations

Avogadro's Number:
$N_A = 6.02 \times 10^{23}$

Boltzmann's Constant:
$k = 1.38 \times 10^{-23}\ J/^oK$

Gas Constant:
$R = N_A k$
$= (6.02 \times 10^{23})\ (1.38 \times 10^{-23}\ J/^oK)$
$= 8.31\ J/^oK$

P = Absolute pressure (in Pa)
V = Volume (in m^3)
N = Number of atoms (unitless)
n = Number of moles (unitless)
T = Kelvin temperature (in ^oK)

Note: $Nk = (nN_A)\ k$
$= n\ (N_A k)$
$= nR$

Without proof, we state below two forms of the Ideal Gas Law

PV = NkT (The N-Form)

PV = nRT (The n-Form)

Related Units: $J/m^3 = (N\text{-}m)/m^3 = N/m^2$
=

Example A:

A container having a fixed volume of 0.04 m^3 is filled with 80 grams of argon (A = 39.95) gas at a temperature of 315 ^oK.

What is the gas pressure?

Solution:

$n = (80\text{ g}) /(39.95\text{ g/mole})$
$= 2.00$ moles

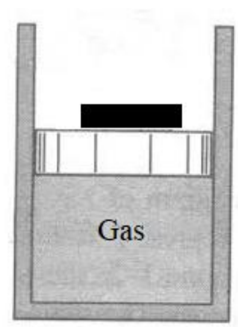
$PV = nRT$
$P = nRT/V$
$= (2.00)\ (8.31 J/^oK)(315\ ^oK)/(0.04\ m^3)$
$= 130{,}882\ J/m^3$
$= 130{,}882$ Pa

Example B:

On top of a weightless, movable piston inside a cylinder is placed a 600-N object. Inside the cylinder is 10 moles of an ideal gas. The temperature of the gas is 500 ^oK, and the area of the piston is 0.06 m^2.

What is the volume of the gas?

The piston is movable, so the piston moves until the pressure forces are balanced, which occurs when the internal pressure is the same as the external pressure, which often is just atmospheric pressure, P_o = 101,000 Pa. In this case, however, the external pressure is the sum of atmospheric pressure at sea-level (101,000 Pa), plus the added pressure due to the 600-N force acting on the cylinder top's 0.06 m^2 area:



$P = P_o + F/A$
$= 101{,}000 + 600 /0.06$
$= 111{,}000$ Pa

Using the ideal gas law, we have
$V = nRT/P$
$= (10)(8.31)(500)/111{,}000$
$= 0.37\ m^3$

The Ratios Equations

(The n-form)

$P_2V_2 = n_2 RT_2$
$P_1V_1 = n_1 RT_1$

Divide to obtain *"The Ratios Equation"*

$(P_2/P_1)(V_2/V_1) = (n_2/n_1)(T_2/T_1)$

The table below lists a few of the "thermodynamic" processes we will encounter in this chapter. Not shown are the constant mass processes in which gas neither enters nor leaves the container, i.e., $n_1 = n_2$. The names by which chemists refer to these processes are included on the table. Student's need not memorize these names.

Isobaric Process	Constant Pressure	$P_2 = P_1$	$(V_2/V_1) = (T_2/T_1)$ "Charles' Law"
Isochoric Process	Constant Volume	$V_2 = V_1$	$(P_2/P_1) = (T_2/T_1)$ "Gay-Lusaac Law"
Isothermal Process	Constant Temperature	$T_2 = T_1$	$(P_2/P_1) = (V_1/V_2)$ "Boyle's Law"

Unless otherwise specified, students should assume that the process is one of constant mass, i.e., the container is unvented—no gas can enter or leave the container.

Example A:

A cylinder containing gas at a temperature $T_1 = 300\ ^oK$ is topped by a movable piston. As the temperature of the gas is changed, the volume is reduced by 20%. What is the new temperature of the gas?

$(P_2/P_1)\ (V_2/V_1) = T_2/T_1$

The piston is movable, so the pressures before and after equal the external pressure, so the ratio of the pressures is 1:

$(0.80) = (T_2 /300)$

$T_2 = 240\ ^oK$

Example B:

Forty moles of an ideal gas in a *vented* cylinder with a movable piston is held at a constant temperature by placing the cylinder in a large container of liquid at a temperature that is not allowed to change.

(a) How many moles of gas will remain if the volume is reduced from 8.0 m^3 to 2.0 m^3?

$(P_2/P_1)\ (V_2/V_1) = (n_2/n_1)\ (T_2/T_1)$ (Equation 1)

The piston is movable, so the ratio of the pressures is 1, and the temperature is constant so the ratio of the temperatures is also 1:

$$V_2/V_1 = (n_2/n_1)$$
$$2/8 = n_2/40$$
$$n_2 = 10 \text{ moles}$$

(b) How many moles left the container?

There initially were 40 moles, but now there are only 10:

$$40 - 10 = 30$$

Thirty moles left the container.

Example A:

An unvented cylinder of gas with an immovable piston contains air at 450 ^oK and a pressure of 300 kPa. The temperature is then increased to 900 ^oK.

What is the new pressure?

Solution:

Recall the ideal gas law applied to an unvented cylinder:

$(P_2/P_1)\ (V_2/V_1) = (T_2/T_1)$

The lid cannot move, so the volume is constant:

$P_2/P_1 = T_2/T_1$
$P_2/300 = 900/450$
$P_2 = 600$ kPa

Example B:

In a constant-temperature ideal gas process of an ideal the volume is reduced to one-third of its previous value. What happens to the pressure?

Solution:

$(P_2/P_1)\ (1/3) = 1$
$P_2/P_1 = 3$
The pressure is tripled.

Kinetic Energy of an Ideal Monatomic Gas

Without proof, we state here that the average kinetic energy *per atom* in an ideal monatomic gas is given below:

$$K = (3/2)\ kT$$

Example:

What is the average speed of the atoms in a gas of argon at 500 ^oK?

First, calculate the mass of an argon atom:

$39.95 / 6.02 \times 10^{23} = 6.64 \times 10^{-23}$ g
$m = 6.64 \times 10^{-26}$ kg

$$\frac{1}{2} mv^2 = (3/2)\, kT$$
$$\frac{1}{2} (6.64 \times 10^{-26})\, v^2 = (3/2)(1.38 \times 10^{-23})\, 500$$

$$v = 558 \text{ m/s}$$

Internal Energy of an Ideal Monatomic Gas

Three Different Equations

The "internal energy" (i.e., thermal energy, heat energy) of N atoms of an ideal monatomic gas is N times the average kinetic energy per atom:

For one atom:
$K = (3/2)kT$

For N atoms:
$$\mathbf{E = (3/2)\, NkT}$$

Earlier, we showed that $Nk = nR$

$$\mathbf{E = (3/2)\, nRT}$$

From the Ideal Gas Law, $nRT = PV$:

$$\mathbf{E = (3/2)\, PV}$$

Note: No matter how the total energy of an ideal gas is calculated, it does *not* depend on which element comprises the gas; it could be, for example, argon, neon, hydrogen, or helium.

Example A:

What is the internal energy of 0.001 m^3 of an ideal gas at pressure 10 P_o?

$$E = (3/2)\,PV$$
$$= (3/2)(10)\,(1.01 \times 10^5)(0.001)$$
$$= 1515\text{ J}$$

Example B:

What is the internal energy of six moles of an ideal monatomic gas at 400 ^oK?

$$E = (3/2)\,nRT$$
$$= (3/2)\,(6)(8.31)(400)$$
$$= 2.99 \times 10^4\text{ J}$$