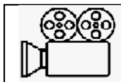
Physics 23 Chapter 13
Thermal Energy Transfer
Dr. Joseph F. Alward

There are three methods by which heat (thermal) energy is transferred from one location to another one:

1. Conduction
2. Convection
3. Radiation

We begin with a discussion of heat conduction.

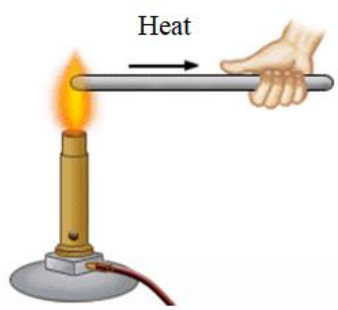
	Heat Conduction Animation

Rapidly moving flame molecules collide with iron atoms, speeding them up, and they, in turn, collide with and speed their neighboring iron atoms. This process by which kinetic energy moves along the bar is similar to the tipping over of a row of dominoes. The name for this process is "heat conduction."

The flow rate at which heat propagates through a substance is proportional to a property called "thermal conductivity."

k = "Thermal Conductivity"
Units: W/(m-C°)

The figure below shows one end of an iron bar in a flame.



Heat Conduction

The figure at the right shows a window of thickness x and cross-sectional area A.

The temperatures of the air on either side differ by the absolute amount shown below:

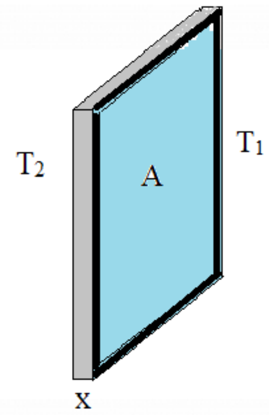
$$|\Delta T| = |T_2 - T_1|$$

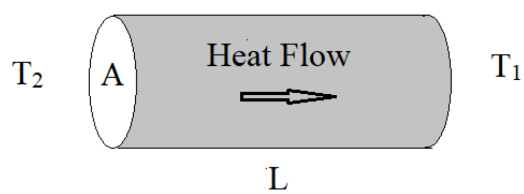
The rate at which thermal energy (heat) is conducted from the hotter side to the cooler side is given by the equation below:

$$R = \frac{kA\,|\Delta T|}{x}$$

If the medium through which thermal energy is traveling is a rod instead of a window, then the thickness x in the equation above is more appropriately replaced by the length L of the rod.

$$R = \frac{kA\,|\Delta T|}{L}$$





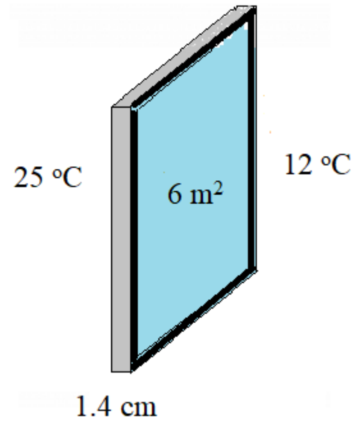
Example A:

The living-room window shown in the figure has a thermal conductivity 0.96 W/m-C^o. The temperature inside the room is 25 ^{o}C; the outside air temperature is 12 ^{o}C.

How much thermal energy will pass into the outdoor air in two hours, assuming the indoor and outdoor temperatures don't change?

$R = (0.96\ W/m\text{-}C^o)\ (6.0\ m^2)\ (13\ C^o)/0.014\ m$
$= 5349\ W$

$Q = (5349\ J/s)(7200\ s)$
$= 3.85 \times 10^7\ J$



Example B:

$R_o = 3600$ W for a cylindrical bar

What would be the rate for a second bar of the same length made of a substance with the same thermal conductivity if its cross-sectional area is half of the other one?

$R = kA\,|\Delta T|\,/\,L$
$R_o = kA_o|\Delta T|\,/\,L$

Divide:
$R/R_o = (A/A_o)$
$= ½$
$R = ½\ R_o$
$= ½\ (3600)$
$= 1800\ W$

Example:

(a) What is the temperature T at the interface of the two layers in the wall at the right? On the left of the wall is bedroom air at 25 ^oC, and on the right is outdoor air at 4 ^oC.

Assume that however fast heat travels through the first layer, the heat, which has nowhere else to go but through the second layer and into the outdoor air, travels at the same rate:

$$R_1 = R_2$$

$0.03\ A\ |T - 25|\ /0.076 = 0.08\ A\ |T - 4|\ /0.019$

Heat flows from left to right, so the temperature at the interface lies between 4 ^oC and 25 ^oC:

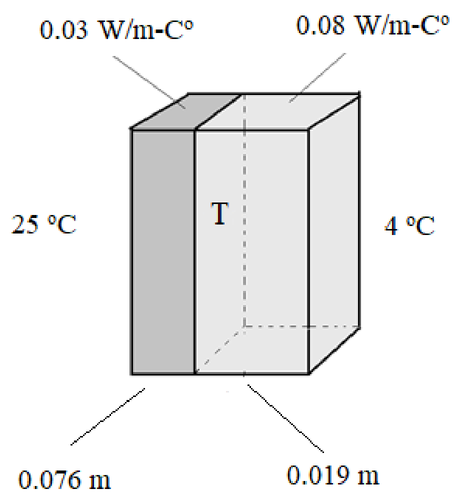
$4\ ^oC < T < 25\ ^oC$

Remove the absolute value signs from the equation above and rewrite the equation while ensuring that the quantities in the parentheses would be positive, consistent with the inequality equation given above:

$0.03\ A\ (25 - T)\ /0.076 = 0.08\ A\ (T - 4)/0.019$

Divide out the areas A and solve for T:

$T = 5.8\ ^oC$



(b) Assume the wall's area is 4.0 m^2. What is the energy flow rate through the left half of the wall?

$R = 0.03\ (4.0)\ (25 - 5.8)/0.076$
$= 30.32\ W$

(c) What is the flow rate through the right half of the wall?

$R = 0.08\ (4.0)\ (5.8 - 4)/0.019$
$= 30.32\ W$

The energy flow rates are "continuous," (the same everywhere) as expected.

Double-Paned Windows

Double-paned windows drastically reduce the amount of heat loss.

Heat through glass travels 48 times more rapidly than it does through air.

Two panes of glass sandwiching air save a lot of money on energy bills. A typical family will see their energy bill reduced by 40% if all the windows in the house are replaced by double-paned ones.

Substance	k (W/m-C^{o})
air	0.02
oak wood	0.17
cement	0.29
water	0.58
asbestos	0.74
window glass	0.96
concrete	1.40
clay	1.80
steel	16
aluminum	205
diamond	1000

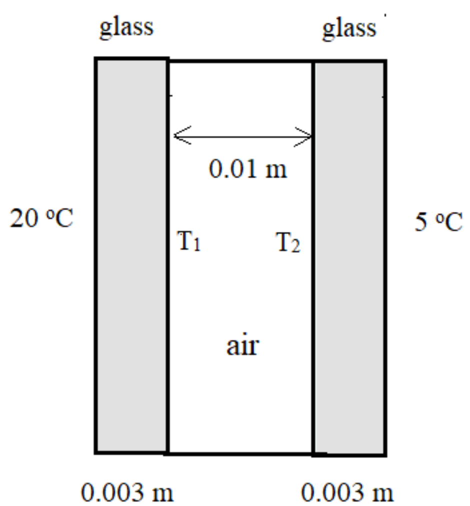
Example:

Calculate the energy flow rate R through the double-pane window below. The window has an area of 4.0 m^2. The temperatures at the two glass-air interfaces are T_1 and T_2.

Note before beginning that T_1 will be less than 20 ^oC, and T_2 will be greater than 5 ^oC.

Glass: $k = 0.96$ W/m-C^o
Air: $k = 0.02$ W/m-C^o

An edge view of the window, not drawn to scale, is shown below.



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Continued from the left....

Energy flow rates are analogous to fluid volume flow rates: They are the same through all cross sections of the medium through which they travel.

The flow rate through the left pane equals the flow rate through the air:

$0.96\ (4.0)(20 - T_1)/0.003 = 0.02\ (4.0)\ (T_1 - T_2)/0.01$

Simplifies to:
$\mathbf{160\ (20 - T_1) = T_1 - T_2}$ (Equation 1)

The flow rate through the air equals the flow rate through the right pane:

$0.02\ (4.0)\ (T_1 - T_2)/0.01 = 0.96\ (4.0)\ (T_2 - 5)/0.003$

Simplifies to:
$\mathbf{(T_1 - T_2) = 160\ (T_2 - 5)}$ (Equation 2)

Solving the two equations for the temperatures, we get

$T_1 = 19.907\ ^oC$
$T_2 = 5.093\ ^oC$

The rate through the left pane:

$R = 0.96\ (4.0)(20 - 19.907)/0.003$
$= 119$ W

The rate through the air:

$R = 0.02\ (4.0)(19.907-5.093)/0.01$
$= 119$ W

The rate through the right pane:

$R = 0.96\ (4.0)\ (5.093-5)/0.003$
$= 119$ W

As expected, the energy flow rate is the same everywhere (continuous).

Humidity

When no more water vapor can be added to the air without it beginning to condense, the air is said to be “saturated.” Saturated air has a “humidity” of 100%. The “dew point” is the temperature at which atmospheric water vapor begins to condense, creating fog or water droplets on leaves and blades of grass.

Let **M** be the number of grams of water vapor per cubic meter that saturates the air. This number depends on the air temperature; the table below illustrates this behavior:

T (^{o}C)	M (g/m^3)
10	9
16	13
17	13.5
21	18
24	22
29	30
32	35

The higher the temperature, the higher-speed air molecules more easily break apart water molecules attempting to bond together. The higher the temperature, the larger the **M**.

Let **m** be the number of grams of water vapor that are present in each cubic meter of air.

The air’s humidity is calculated according the equation below:

$$H = (m/M)\ 100\%$$

Condensation on Leaves

Example:

In Sacramento at 7:00 pm one evening a meteorologist announced that the temperature was 21 ^{o}C, and the humidity was 75%.

(a) How many grams of water vapor per cubic meter were in the air at that time?

Solution:

According to the table at the left, at 21 ^{o}C, M =18 g/m^3.

$$H = (m/M)\ 100\%$$
$$75\ \% = (m/18)\ 100\%$$
$$m = 13.5\ \text{g/m}^3$$

(b) What did the she say was the “dew point?”

Answer: 17 ^{o}C

Example:

In the valley one morning the temperature was 29 ^oC, and the humidity was 53%.

(a) How many grams of water vapor per cubic meter were there at that time?

T (^oC)	M (g/m^3)
10	9
16	13
21	18
24	22
29	**30**
32	35

$$53\% = (m/30)\ 100\%$$
$$m = 15.9\ g/m^3$$

(b) Two hours later the temperature fell to 24 ^oC. What was the new humidity?

$H = (15.9/22.0)\ 100\%$
$\quad = 72.3\%$

(c) Later, moist air swept down from the mountains into the valley adds 11.0 grams of water vapor to each cubic meter of air.

What then was the new humidity?

The additional water vapor raised the total to

$15.9 + 11.0 = 26.9\ g/m^3$

22 g/m^3 saturates the air, so 4.9 grams condense out of each cubic meter of air, leaving behind completely saturated air and a humidity of 100%.

Calculation of Saturation Mass

The following equation permits a calculation of the saturation mass per cubic meter for a given Celsius temperature T:

$$M = 0.032\ T^2 - 0.17\ T + 7.49$$

This equation may be used on exams whenever the table of saturation masses is not provided.

<u>Example</u>:

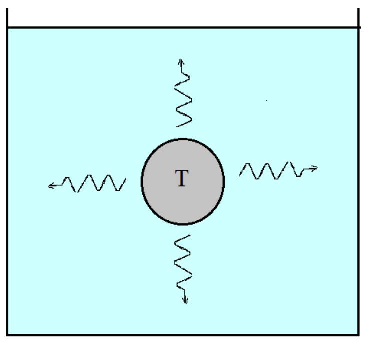
At noon, Sunday, 7.5 grams of water vapor were present per cubic meter of air, the humidity was 43 %. What was the temperature (in ^{o}C)?

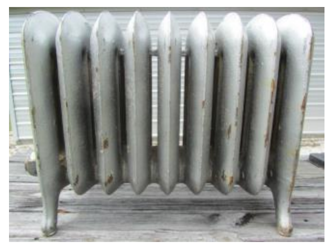
Solution:

$7.5/M = 0.43$
$M = 17.44$ g
$17.44 = 0.032\ T^2 - 0.17\ T + 7.49$
$T = 20.49\ ^{o}C$

Infrared Radiation

All objects emit some of their heat as “infrared” (IR) radiation (sometimes called, “heat rays”), a form of electromagnetic energy not visible to humans.





Old-fashion radiator filled with steam radiates IR to heat the room

The rate at which thermal energy is radiated away is symbolized as R, and its units are joules per second (watts, W).

$R = \sigma e A T^4$

σ = Stefan’s Constant
= 5.67×10^{-8} W/m^2-^{o}K^4

e = emissivity
A = Surface Area
T = Kelvin temperature

Note: We use the same symbol (T) for Kelvin temperature instead of T_K, in order that the equations we will deal with are less cluttered. There will be no possibility of ambiguity involving the Celsius temperature symbol (T) because Celsius temperatures play no role in our heat transfer discussions in this chapter.

Example A:

The surface of a sphere of radius 1.2 m has an emissivity e = 0.80 and a temperature of 600 ^{o}K.

At what rate is thermal energy being radiated?

$A = 4\pi\,(1.2)^2$
$\quad = 18.1\ m^2$

$R = (5.67 \times 10^{-8}\ W/m^2\text{-}^{o}K^4)(0.80)\ (18.1\ m^2)\ (600\ ^{o}K)^4$
$\quad = 1.06 \times 10^5\ W$

Example B:

A cube whose side-lengths are 8.0 cm has an emissivity of 0.90, and is radiating 3000 watts of IR. What is the Kelvin temperature of the cube's surface?

A cube has six square faces:

$A = 6\ (0.08\ cm)^2$
$\quad = 3.84\ (cm)^2$
$\quad = 3.84\ (1 \times 10^{-2}\ m)^2$
$\quad = 3.84 \times 10^{-4}\ m^2$

$3000 = (5.67 \times 10^{-8})(0.90)(\ 3.84 \times 10^{-4})\ T^4$
$\quad T = 3518\ ^{o}K$

Example A:

Use the following data to estimate the radius of the Sun:

Surface Temperature: 5778 ^oK
Emissivity: 1.0
Radiation Rate: 3.8×10^{26} W

$$R = \sigma e A T^4$$

$$3.8 \times 10^{26} = (5.67 \times 10^{-8})(1.0)(4\pi r^2)(5778)^4$$

$$r = 6.93 \times 10^8 \text{ m}$$

Actual Radius: $r = 6.96 \times 10^8$ m

Example B :

The IR radiation rate from an object is 2000 W. What would be the new rate if the Kelvin temperature is doubled and, as the temperature rises, the object's area expands to an area 15% greater?

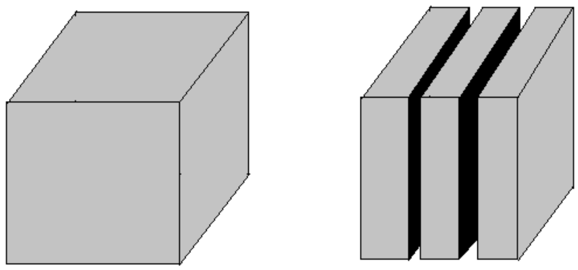
$A = 1.15\ A_o$
$T = 2\ T_o$
$R = \sigma e A T^4$
$R_o = \sigma e A_o T_o^4$

$$\begin{aligned} R/R_o &= (AT^4)/(A_oT_o^4) \\ &= (A/A_o)(T/T_o)^4 \\ &= (1.15)(2)^4 \\ &= 18.40 \end{aligned}$$

$$\begin{aligned} R &= 18.40\ R_o \\ &= (18.40)(2000) \\ &= 36{,}800 \text{ W} \end{aligned}$$

Example:

A cubic block of side-length L radiating 2400 watts is sliced into three equal rectangular blocks.



$A_o = 6L^2$ $A = 10\ L^2$

What is the new total rate of radiation?

Before cutting, the total surface area was six times the area L^2 of a square face. Each cut shown above opens up two additional faces: two cuts opens up four more faces for a total of ten faces.

$R = \sigma e A T^4$
$R_o = \sigma e A_o T_o^4$

$R/R_o = A/A_o$
$= (10\ L^2)/(6\ L^2)$
$= 10/6$

$R = (10/6)\ R_o$
$= (10/6)\ 2400$
$= 4000$ W

Example:

A sphere radiating 6000 W is cut in half. What is the new rate of emission?

Note: Cutting the sphere in half creates two hemispheres, each of which adds a circular πr^2 area .

$A_o = 4\pi r^2$

$A = 4\pi r^2 + \pi r^2 + \pi r^2$

$\quad = 6\pi r^2$

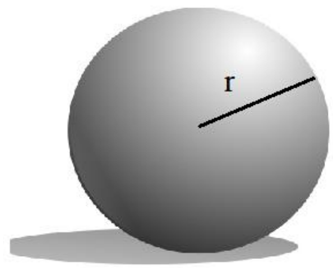
$R/R_o = A/A_o$

$\quad = 6/4$

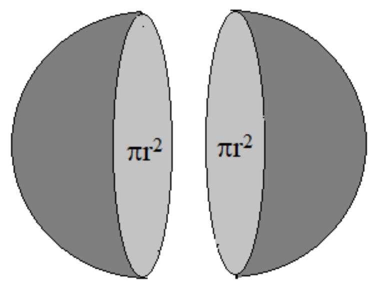
$R = (6/4)\, R_o$

$\quad = (6/4)\, 6000$

$\quad = 9000$ W



$A_o = 4\pi\, r^2$

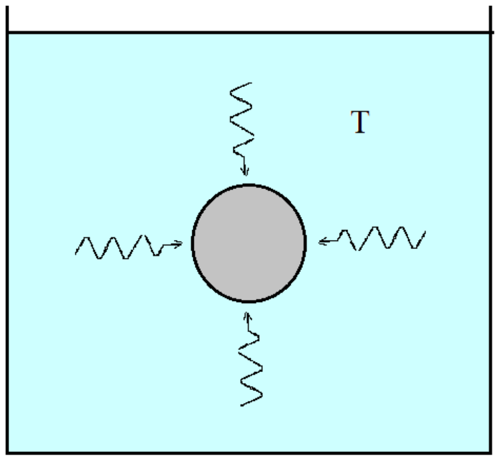


$A = 6\pi r^2$

Absorbing Infrared from Surrounding Fluids

Objects not only radiate IR, but also *absorb* IR radiated from other objects, including fluids in which they might be immersed. The equation for the rate of absorption of IR from fluids in which the object is immersed is the same as the one for emission:

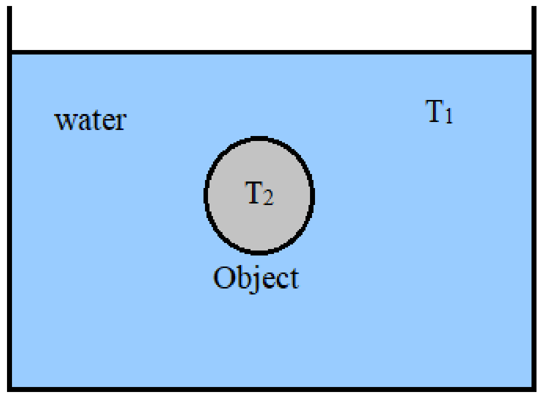
The diagram below illustrates the absorption of IR from surrounding fluid; not shown is the IR that is radiated away from the object.



R = Rate of Absorption of IR

$R = \sigma e A T^4$

Net Gain or Loss of IR Energy



T_1 = Fluid temperature
T_2 = Object temperature

R_{net} = Rate of emission - Rate of absorption
$= \sigma e A T_2^{\,4} - \sigma e A T_1^{\,4}$
$= \sigma e A\,(T_2^{\,4} - T_1^{\,4})$

If R_{net} is positive, more energy flows away from the object than flows in, so there will be a net loss of energy by the object. The example below illustrates this.

<u>Example</u>:

A swimmer whose skin temperature is $T_2 = 303\ ^oK$ is immersed in lake water whose temperature is $T_1 = 280\ ^oK$. Her surface area is 1.2 m^2 and her emissivity is 0.70.

What is this person's net rate of IR energy loss?

$$
\begin{aligned}
R_{net} &= \sigma e A\,(T_2^{\,4} - T_1^{4}) \\
&= 5.67 \times 10^{-8}\,(0.70)(1.2)\,(303^4 - 280^4) \\
&= 109 \text{ watts}
\end{aligned}
$$

R_{net} is positive, which means that the swimmer is suffering a net *loss* of heat energy: The swimmer is radiating more energy each second than she absorbs. If it were not for her body's metabolism, which creates the energy necessary for cell repair and other bodily activities, as well as generates heat, this person would soon be as cold as her surroundings.

Convection

Convection currents transfer heat from one place to another by bulk movement of a fluid --such as water or air--caused by density differences within the fluid, which causes less dense portions of the fluid to be buoyed upward while more dense portions sink.

Example:

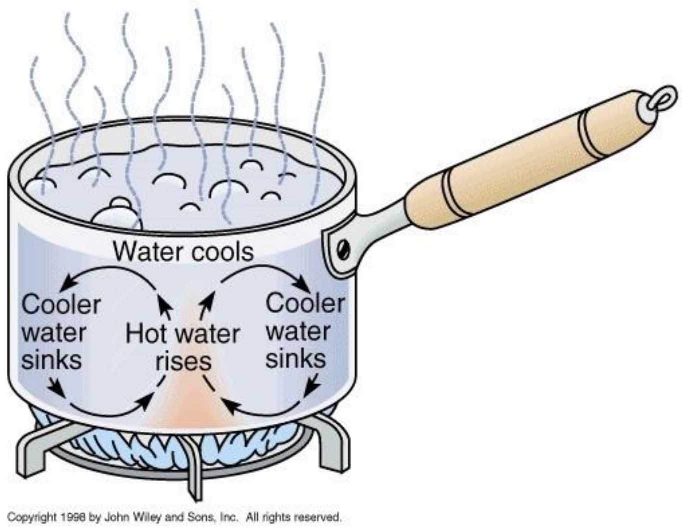
Cooled fluids contract and become denser, and sink. Heated fluids expand, become less dense and rise.

Water at the bottom of the pot is heated, expands, becomes less dense than the surrounding cooler water, and is buoyed to the top.

The risen hot water at the top is cooled by the kitchen air, becomes more dense than the surrounding hotter water below, loses its buoyancy and sinks.

The process is repeated until all of the water is at the same temperature and no more density differences exist, and no more rising or falling of water occurs. This occurs at the temperature below:

$100\ ^{o}$C



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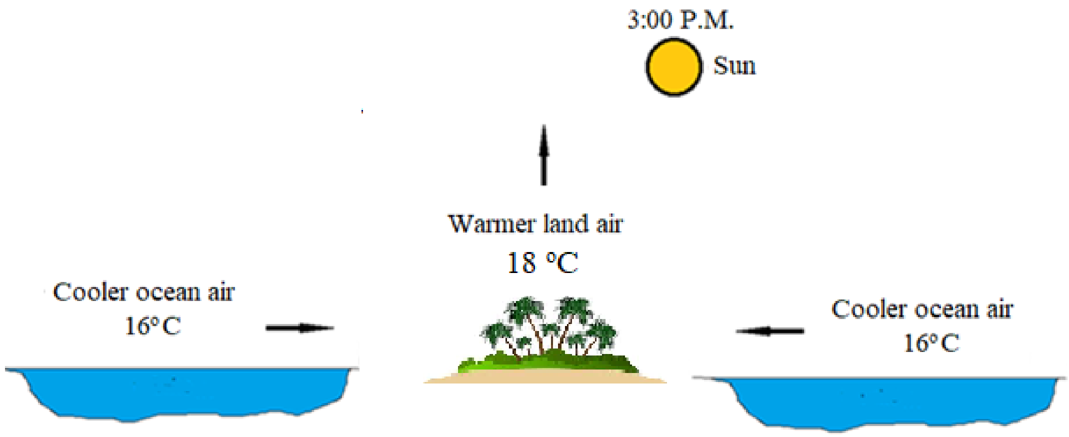
Convection Current Example: Ocean Breezes

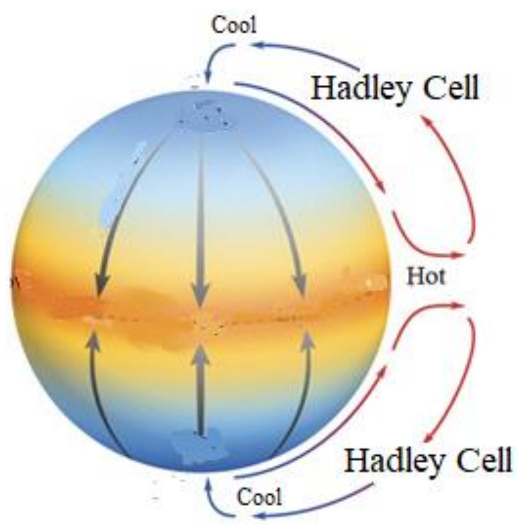


Recall the example of land surrounded by ocean water discussed in Chapter 12: Land has a lower heat capacity than water, so its temperature increases more rapidly than water's when heat is gained or lost. Therefore, during the day sunlight increases land's temperature more than the lake's temperature. Cooler ocean air flows toward land to replace rising warmer, less dense air.

Convection Current Example: Hadley Cells

Precisely analogous to the ocean breeze example above, hot air above the equator rises and is replaced by cooler air rushing in from the poles.

These circulation "cells" are called "Hadley cells," named after George Hadley (1685-1768), an English lawyer and amateur meteorologist.



Humidity

Humidity is a measure of the amount of water vapor in the air. When no more water vapor can exist in the air without it condensing, the air is said to be "saturated" with water vapor.

The percentage of the maximum water vapor that is allowed is the humidity.

Let M = Maximum Mass of Water Vapor
Allowed per Cubic Meter
(the saturation mass)

Let m = Actual Mass

$$H = (m/M)\ 100\%$$

Condensation on Leaves

The saturation mass M depends on the air temperature. As the table below shows, the higher the temperature, the greater is the saturation mass.

T (^oC)	M (g/m^3)
10	9
16	13
21	18
24	22
29	30
32	35

Example:

In Sacramento at 7:00 pm one evening the temperature was 21 ^oC, and the humidity was 75%.

How many grams (m) of water vapor were there per cubic meter?

Solution:
From the table, we see that at 21 ^oC, M = 18 g

$H = (m/M)\ 100\ \%$
$75 = (m/18)\ 100$
$m = 13.5\ g/m^3$

Example:

In the valley one morning the temperature was 29 ^oC, and the humidity was 53%.

First, let's determine the value of M at 29 ^oC from the table:

M = 30 g

(a) How many grams (m) of water vapor were there per cubic meter?

53% = (m/30) 100%
m = 15.9 g/m^3

(b) Two hours later the temperature fell to 24 ^oC. What was the new humidity?

The value of m is still m = 15.9 g, but the saturation mass changes to 22 g at 24 ^oC.

H = (15.9/22.0) 100%
= 72.3%

T (^oC)	M (g/m^3)
10	9
16	13
21	18
24	22
29	30
32	35

Example:

(a) At 1:00 pm in the valley the air temperature was 16 ^oC and the humidity was 70%. How many grams of water vapor were there in each cubic meter of air at that time?

70 = (m/13) 100
m = 9.1 g

(b) At 7:00 pm the temperature had risen to 21 ^oC, and during the previous six hours moist wind had swept 2.3 grams of water vapor per cubic meter into the valley. What was the humidity then?

The new saturation mass is M = 18 g.

Added to the initial 9.1 g is 2.3 more grams:

m = 9.1 + 2.3
 = 11.4 g

H = (11.4 /18) x 100%
 = 63%