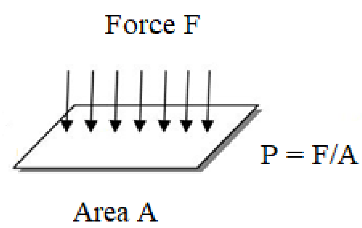
# Physics 23 Chapter 11

## Principles of Archimedes, Pascal, and Bernoulli

Dr. Joseph F. Alward

| | |
|---|---|
| The pressure that is applied over an area equals the force applied, divided by the area.<br><br><br><br>Pressure Units:<br><br>1.0 “pascal” (Pa) = 1.0 $N/m^2$ | <u>Example</u>:<br><br>A block weighing 4000-N, whose base area is 2.0 $m^2$, rests on the ground. What pressure does this portion of the ground experience?<br><br>F = 4000 N<br>A = 2.0 $m^2$<br><br>P = F/A<br>= (4000 N)/ (2.0 $m^2$)<br>= 2000 $N/m^2$<br>= 2000 Pa<br><br>This pressure is “uni-directional, “ meaning that its effect is felt in only one direction. In this case, downward. |

Example:

A 100-lb (445 N) person wearing stiletto heels, standing on one leg, rocks back onto the heel. The base of the heel is circular, with a radius of 0.20 cm.


What pressure does the heel exert on the floor?

$A = \pi\,(0.20 \times 10^{-2}\ m)^2$
$\quad = 1.26 \times 10^{-5}\ m^2$
$P = F/A$
$\quad = 445\ N / (1.26 \times 10^{-5}\ m^2)$
$\quad = 3.54 \times 10^{7}\ N/m^2$
$\quad = 3.54 \times 10^{7}\ Pa$

Like the block on the tabletop above, heel pressure is uni-directional.

## Atmospheric Pressure

Earth's atmosphere is an approximately 40-mile thick blanket of oxygen, nitrogen, and other gases weighing $5.17 \times 10^{19}$ N. This "blanket" of air rests on Earth's surface.

What is the uni-directional pressure the atmosphere exerts on the ground?

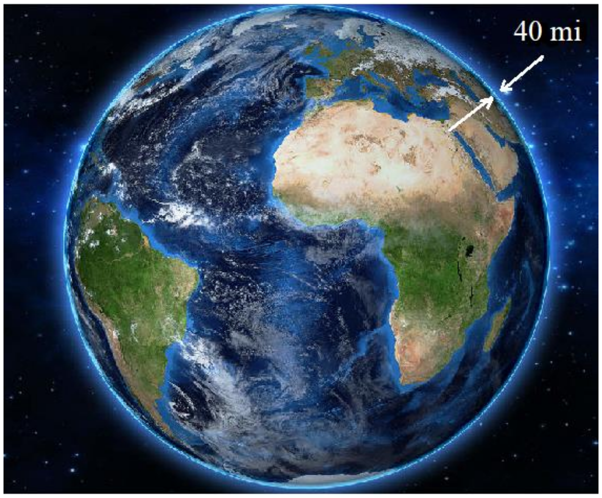
$F = 5.17 \times 10^{19}$ N
$A = 4\pi R^2$ (surface area of a sphere)
$= 4\pi (6.38 \times 10^6)^2$
$= 5.12 \times 10^{14}\ m^2$

$P = F/A$
$= 5.17 \times 10^{19}\ N / 5.12 \times 10^{14}\ m^2$
$= 101{,}000\ N/m^2$

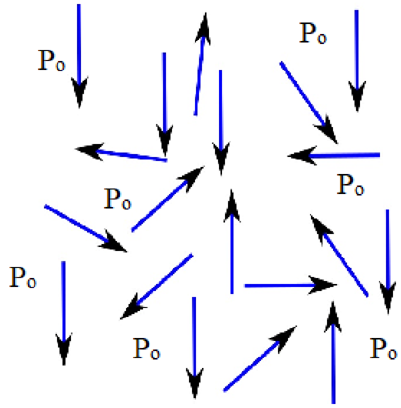
We will use the symbol "$P_o$" to represent atmospheric pressure at "sea-level." The subscript reminds us that the location in question is at zero elevation.

$P_o = 101{,}000$ Pa

Unlike the pressure exerted on the ground by the 4000-N block resting on the ground studied earlier--a pressure that is uni-directional, air pressure exerts itself not only uni-directionally, as far as the ground is concerned, but in fact at points within the atmosphere, the air pressure acts *equally in all directions*, as indicated in the figure at the right; it's said to be "omnidirectional."




Atmospheric pressure is omni-directional.



# Non-Standard Air Pressure Units

As noted earlier, the SI units of air pressure are pascals.  Other, non-standard units are given below.

From the Greek word, “*baros*,” meaning “weight, “ and reminding you of “barometer,” we have the following non-standard unit of pressure:  the “millibar,” (mb)

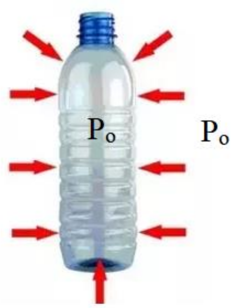
1.0 “millibar” (mb) = 100 Pa
$P_o$= 1010 mb

Another non-standard unit is the lbs/in$^2$:

$P_o$ = 14.7 pounds per square inch (psi)

Air pressure is omnidirectional. As shown in the figure below, there is a 14.7 lbs/in$^2$ air pressure acting in all directions on the inside and outside of the *capless*, empty water bottle.

Atmospheric pressure doesn't crush the bottle because the air pressure inside exerting a force outward is matched by the same air pressure outside exerting a force inward.
If the air were removed from inside, the bottle would collapse.




Not shown above are the balancing air pressure forces inside the bottle.

Try this experiment:

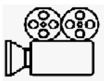
Put the cap on an empty plastic water bottle and put it in the freezer.

The pressure inside the bottle will initially be the same as the pressure outside. However, the freezer will cause the air trapped inside the bottle to cool, which causes the air pressure inside the bottle to decrease.* The greater external pressure defeats the lower air pressure inside the bottle, which causes a partial collapsing of the bottle.

Remove the cap, air rushes in, the pressure inside becomes the same as that outside, and the bottle's shape is partially restored.

*Guy-Lussac's Law: The pressure in a gas of fixed volume decreases when the temperature decreases.

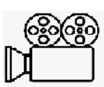
# Railroad Tank Car Imploded



Click here to see what happened to a railroad tank car after it was steam-cleaned inside.

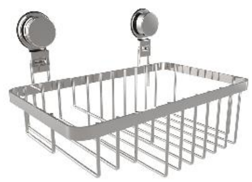
Upon finishing the cleaning, the worker then unthinkingly sealed the tank containing the hot steam, and no air.  While very hot, the steam exerted a pressure against the inside walls of the tank to prevent it from collapsing under the comparatively immense pressure due to the outside atmosphere.

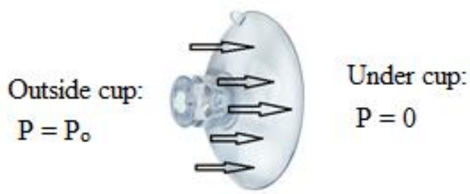
As the tank and water vapor inside cooled overnight, the vapor's pressure decreased (Guy-Lusaac's Law).  The steam eventually condensed, leaving a vacuum inside.  The pressure-less interior is unable to compete against the air pressure of 14.7 pounds per square inch outside, and the tank eventually dramatically collapsed.



Click here to see a vacuum *deliberately* created inside a tank car using a vacuum pump.

# Shower Caddy






Pushing the cup against the smooth shower stall tiles squeezes the air out from underneath. Outside the cup the shower-stall pressure is $P_o$, while the air pressure under the cup is zero.

<u>Example</u>:

Calculate the force in pounds due to atmospheric pressure on one of the shower caddy “suction cups,” assuming the cups are circular, and have a radius of 0.70 inch.

Recall: $P_o = 14.7 \text{ lbs/in}^2$

$F = PA$

$= 14.7 \ \pi(0.70)^2$
$= 22.6$ lbs

# Absolute Pressure vs Gauge Pressure


Air pressure gauges measure how much *greater* the tire's air pressure is compared to the outside atmospheric air pressure.

After some air is added to the "flat" tire, the total air pressure inside is now greater than $P_o$, by a certain amount. That additional pressure is what is displayed on the gauge, and is what we call the "gauge" pressure.

Gauge Pressure = Total Pressure – Atmospheric Pressure

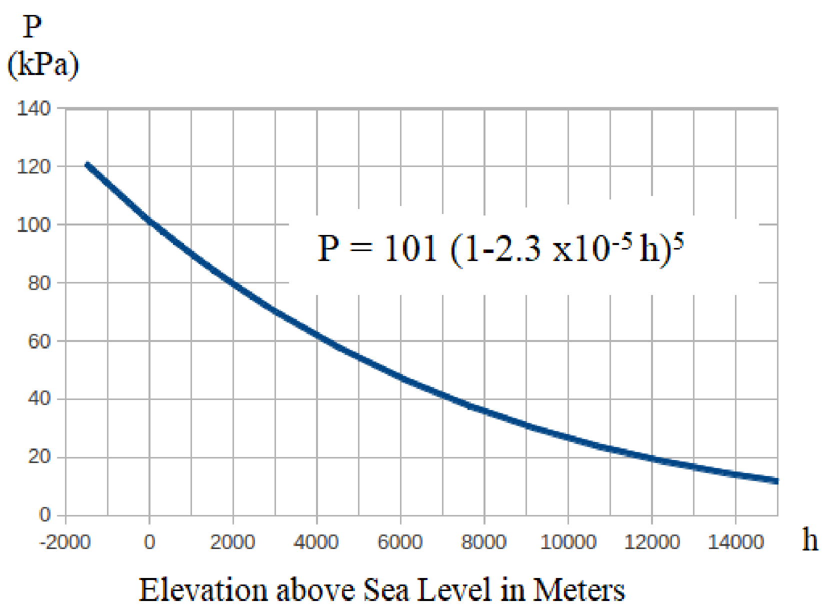
"Total" pressure is sometimes called, "absolute pressure."

| Example A: | Example B: |
|---|---|
| The total air pressure P inside an inflated tire is 50.0 psi.<br><br>What is the gauge pressure of the air inside the tire?<br><br>Gauge Pressure = $P - P_o$<br>$= 50.0 - 14.7$<br>$= 35.3$ psi | What must the gauge pressure in a tire be in order that the total pressure in it be 40 psi? |

## Atmospheric Pressure vs Elevation

The pressure at any elevation depends on the weight of the blanked of air above that elevation. Points on Earth at sea-level have about 40 miles of air piled above it, whereas locations near the top of Mt. Everest have only a 35-mile thick blanket of air above. Consequently, air pressure at the top of that mountain is considerably less than the air pressure at sea-level.

Below is a graph and equation which shows how atmospheric pressure varies with elevation. Note that air pressure is measured in kilo-pascals (kPa), where 1.0 kPa = 1000 Pa.




Elevations:

Dead Sea: -413 meters
Stockton, California: 4 meters

Example A:

What is atmospheric pressure 2000 meters above sea-level?

$P = 101\ (1 - 2.3 \times 10^{-5} \times 2000)^5$
$= 79.8$ kPa

Example B:

At what altitude is atmospheric pressure 80 kPa?

$80 = 101\ (1 - 2.3 \times 10^{-5}\, h)^5$

$(80/101)^{1/5} = 1 - 2.3 \times 10^{-5}\, h$
$h = 1980$ m

# Density

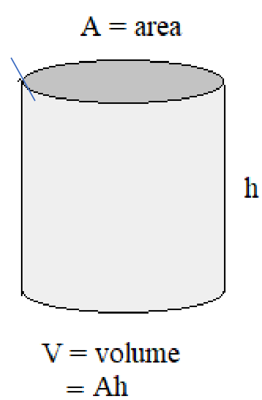
The density of a substance is its mass per volume. The standard units are kilograms per cubic meter ($kg/m^3$).

$$\rho = m/V$$

| Substance | $\rho$ ($kg/m^3$) | $\rho$ ($g/cm^3$) |
|---|---|---|
| Gold | 19,300 | 19.30 |
| Water | 1,000 | 1.00 |
| Ice | 920 | 0.92 |
| Wood | 800 | 0.80 |
| Air | 1.2 | 0.0012 |

The density in units of $g/cm^3$ equals the density in $kg/m^3$, divided by 1000.

Example:




What is the mass of the water inside a cylindrical barrel whose height is 1.7 meters and whose top and bottom areas are 0.25 $m^2$?

$V = Ah$
$= 0.25\ (1.7)$
$= 0.425\ m^3$

$m = (1000\ kg/m^3)\ (0.425\ m^3)$
$= 425$ kg

Example:


Gold sold for $60 per gram on May 22, 2022.

What would have to have been the radius of a solid gold sphere for it to have been worth $10,000 on that day?

Solution:

m = (10,000 dollars)/(60 dollars/gram)
= 166.67 g

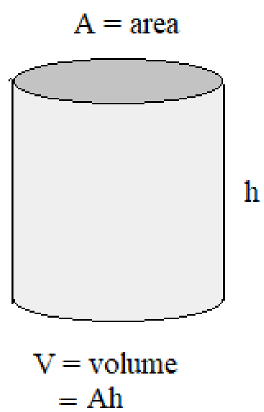
$(19.3\ \text{g/cm}^3)\ (4/3)\ \pi r^3 = 166.67\ \text{g}$
$r = 1.27\ \text{cm}$

Diameter: 2.54 cm (about one inch)

# Water Pressure

Example A:

The cylinder below is filled with water. What is the water pressure on the base?




$$F = mg$$
$$= \rho\,(V)\,g$$
$$= \rho\,(Ah)\,g$$

$$P = F/A$$
$$= \rho(Ah)g/A$$
$$= \rho gh$$

$$= 1000\,(9.8)\,h$$
$$= 9800\,h$$

Example B:

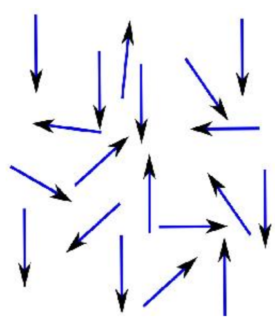
(a) What is the water pressure at a depth d of 10 meters?

$$P = 9800\,h$$
$$= 9800\,d$$
$$= 9800\ (10)$$
$$= 98{,}000\ \text{Pa}$$

(b) What is the total (the absolute) *fluid* pressure--air pressure plus water pressure--at that location?

$$P = 101{,}000 + 98{,}000$$
$$= 199{,}000\ \text{Pa}$$

Water pressure--like air pressure--is omni-directional: its effect is felt in all directions equally: upward, downward, sideways, forward, backward, as indicated in the figure below.



<u>Example</u>:

The “crush” water pressure for World War II German submarines (“U-boats”) was about $3.0 \times 10^6$ Pa. How far below the surface could these U-Boats dive?

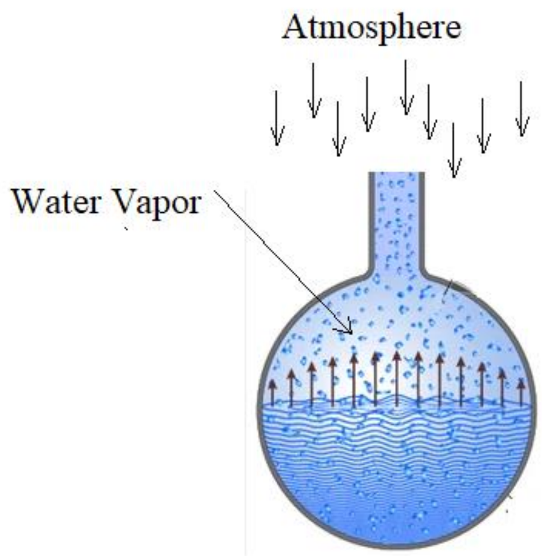
German U-Boat, North Sea, 1940.

Let d = depth of water

$9800\ d = 3.0 \times 10^6$

$d = 306$ m

# Vapor Pressure




As temperature rises, the rate at which water vapor molecules leave the surface increases, which leads to a greater water vapor pressure just above the surface. When the water vapor pressure exceeds the atmospheric pressure, boiling occurs. Water vapor carrying heat away from the water into the atmosphere is matched by the rate at which heat enters the water. The temperature reaches a value that doesn't change while boiling occurs. This temperature—called the "boiling point"-- is 100 $^{o}$C (212 $^{o}$F).

The lower atmospheric pressure is, the lower is the boiling point. Since atmospheric pressure decreases with increasing elevation, the boiling point decreases with altitude.

# Boiling Point versus Atmospheric Pressure

Boiling occurs when the water molecules at the surface have sufficient kinetic energy to break the bonds that attach them to the water's surface and escape into the air.

Also competing against this escape are air molecules that collide with any hopeful escapers and drive them back inside down into the water.

The greater is the atmospheric pressure, the greater must be the kinetic energy of the water molecules (i.e., the hotter the water must be) for the water molecules to escape.

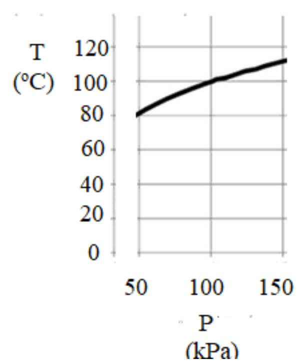
At lower atmospheric pressure, escape and boiling can occur with lower water molecule speeds and therefore lower water temperature.

Click link below to see water boil in a vacuum.

Video

The figure at the right shows how the boiling point of water depends on atmospheric pressure.

Boiling Point vs. Pressure




The equation below is an accurate representation of the graph above.

$$T = 71.6 + (7/25)\,P$$

Example:

At what air pressure will the boiling point of water be 90 $^oC$?

$90 = 71.6 + (7/25)\ P$
$P = 65.7$ kPa

(b) What altitude is this?
$P = 101\ (1 - 2.3 \times 10^{-5}\ h)^5$
$65.7 = 101\ (1 - 2.3 \times 10^{-5}\ h)^5$
$h = 3583$ m

## Buoyancy

Recall that water pressure increases with depth d according to the equation below:

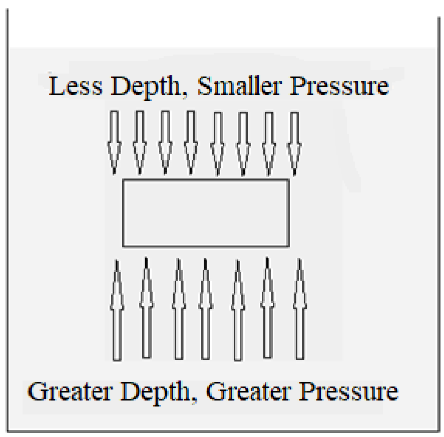
$P = 9800\ d$

(Originally, we presented the water pressure equation in terms of the height h of the water above the submerged object: $P = 9800\ h$.)

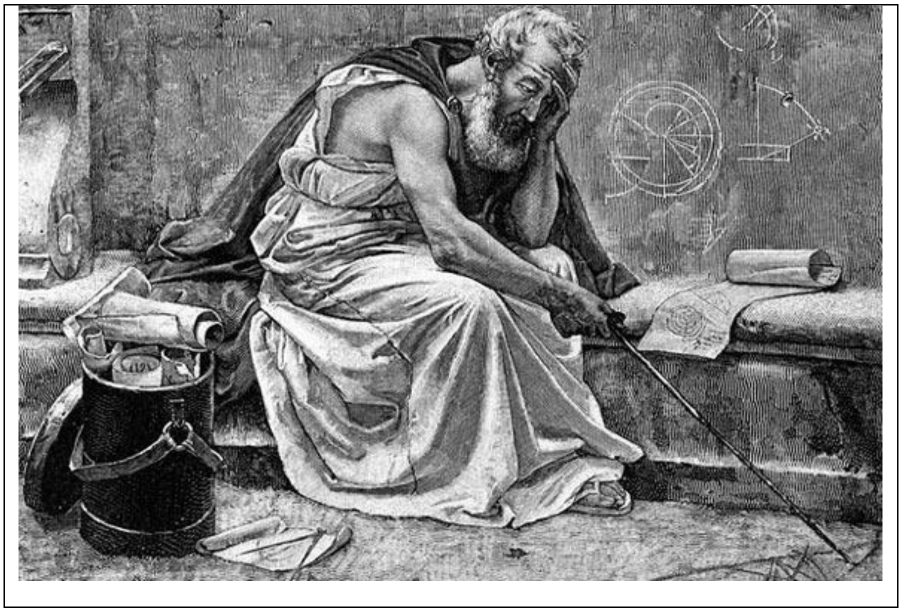
The underside of the block immersed in water at the right is at a greater depth below the water's surface than is the upper side, so the water pressure under the block is greater than the water pressure above it.

The upward force is therefore greater than the downward force. The difference between these two pressure forces is the buoyant force, B.

We will show next that B is the weight of the water displaced.



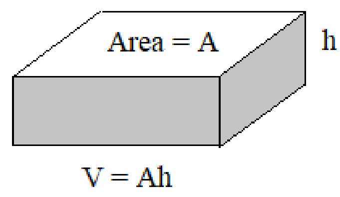
# Archimedes' Principle



Archimedes, inventor, mathematician
287 - 211 BCE

*"The buoyant force acting on an object in a fluid is the weight of the fluid displaced."*

## Preliminary to Proof of Archimedes' Principle




The block above is immersed in a fluid, whose density is ρ. What will be the weight of the fluid displaced by the block?

Whatever is the volume of the block…that's the same as the volume of the fluid displaced to make room for the block, so....

Volume of fluid displaced = Volume of block

$$V = \boxed{Ah}$$

Mass of fuid displaced: $m = \rho V$

$$= \boxed{\rho Ah}$$

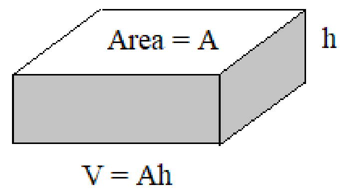
Weight of fluid: $w = mg$

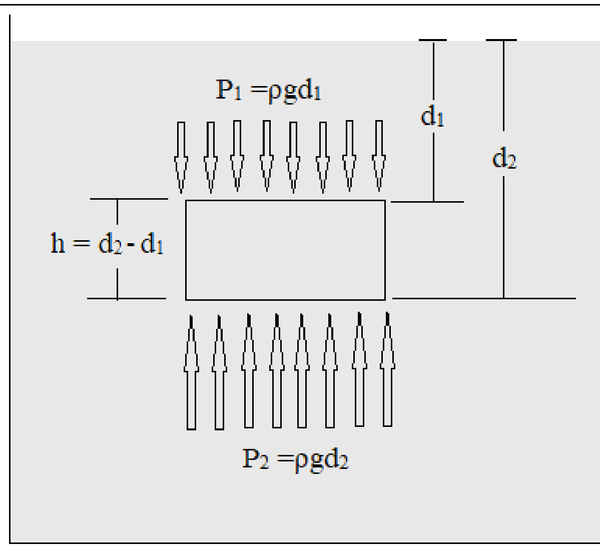
$$= (\rho Ah)g$$

This weight of fluid expression will be used later in our proof of Archimedes' Principle.

# Proof of Archimedes' Principle

Calculate the buoyant force acting on the block discussed in the previous example.




Upward Force: $F_2 = P_2A$
Downward Force: $F_1 = P_1A$

B = Net Fluid Pressure Force

$$\begin{aligned} &= P_2A - P_1A \\ &= A(P_2 - P_1) \\ &= A(\rho d_2 - \rho d_1)\, g \\ &= \rho\ A(d_2 - d_1)\, g \\ &= \rho\ Ah\ g \end{aligned}$$

As shown on the previous page, the quantity above is the weight of fluid displaced.

Archimedes' Principle:

$B = \rho\ Ah\ g$

= weight of fluid displaced

*"The buoyant force B acting on an object equals the weight of the fluid displaced by the object."*

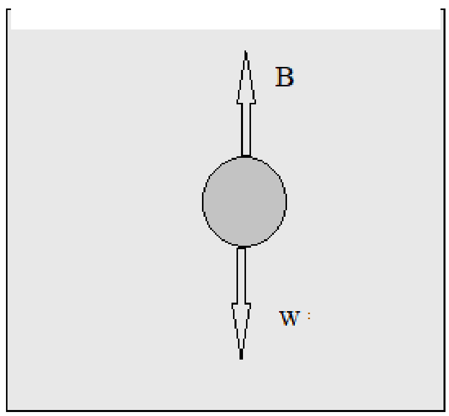
The proof of Archimedes' Principle was given above for objects *completely* under water. We state here without proof that the buoyant force acting on objects that are part in and part out of the water is likewise equal to the weight of the water displaced. In such a case, of course, the volume of water displaced will not be the same as the volume of the object.

# Archimedes' Buoyancy Equation

Note: If V is the volume of water displaced, then its weight is

$$\begin{aligned} w &= \rho V g \\ &= (\rho g)\, V \\ &= (1000 \text{ x } 9.8)\, V \\ &= 9800\, V \end{aligned}$$

$$\boxed{B = (9800 \text{ N/m}^3)\, V}$$

| | |
|---|---|
| <u>Example</u>:<br><br>A solid metal ball whose density is 2800 kg/$m^3$ and whose radius is 0.30 m is placed in a tank of water, and sinks.<br><br><br><br>Volume of Water = Volume of Ball<br><br>$V = (4/3)\pi\, (0.30)^3$<br>$= 0.1131 \text{ m}^3$ | (a) What is the buoyant force on the ball?<br><br>$B = 9800\, V$<br>$= 9800\, (0.1131)$<br>$= 1108$ N<br><br>(b) What is the weight of the ball?<br><br>$w = mg$<br>$= (\rho)(V)\, g$<br>$= (2800)\, (0.1131)\, 9.8$<br>$= 3103$ N<br><br>(c) What is the net force on the ball?<br><br>$F = B - w$<br>$= 1108 - 3103$<br>$= -1995$ N<br><br>The negative sign indicates that the metal ball is accelerating in the negative direction, i.e., it's sinking. |

Example A: A block weighing 200 N is floating in water. What is the weight of the water displaced?

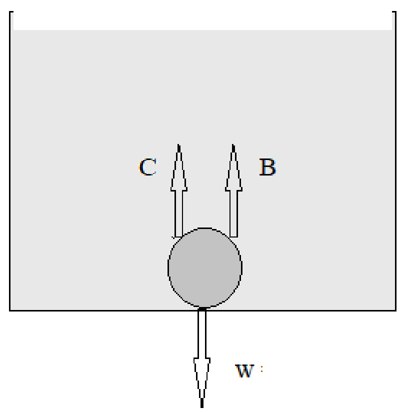
Answer:

A floating object is not accelerating, so the net force on it is zero. Thus, the pull down by Earth (200 N) is balanced by an upward 200-N buoyant force.

By Archimedes' Principle, the weight of the water displaced equals the buoyant force: 200 N.

Example B:

A ball weighing 1200 N sinks to the bottom of a tank of water. The buoyant force it experiences is 700 N. What is the contact force between the bottom of the tank and the ball?



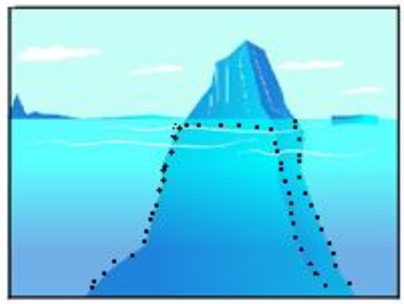

The ball is stationary at the bottom of the tank, so the sum of the vertical components of the forces is zero:

$$C + B - mg = 0$$
$$C = mg - B$$
$$= 1200 - 700$$
$$= 500 \text{ N}$$

# The “Tip of the Iceberg”

Concerning the fraction of a floating object that is under water, we state without proof:

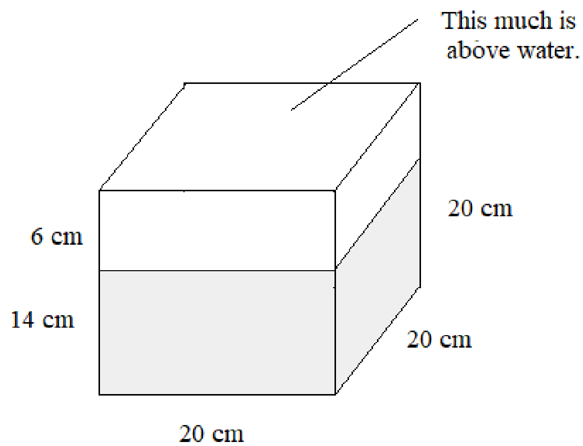
Fraction Below Water = Density of Object/Density of Water

| | |
|---|---|
| <u>Example</u>:<br><br>Sea water has a density of 1029 kg/m$^3$. Ice has a density of 920 kg/m$^3$.<br><br>What fraction of an iceberg is below the water line?<br><br>Fraction Below:<br>= 920/1029<br>= 0.89<br><br>Fraction above: 0.11.<br><br>Only the tip (11%) of an iceberg is visible. |  |

<u>Example</u>:

The cubic object whose side lengths are 20 cm is floating in water. The bottom 14 m is under water.

What is the object's density?




The bottom 14 cm of the 20-cm tall block is under water, so 70% of the block is under water (f = 0.70).

Density of Object/ 1000 = 0.70

Density of Object = 700 kg/m$^3$

Example:

In previous buoyancy problems the fluid displaced was water. In this example, the fluid displaced is air, so the buoyant force equals the weight of the *air* displaced.

The mass of a spherical hot air balloon and its basket and passenger (including the air inside the balloon) is 500 kg. Air density is 1.25 kg/m$^3$. The balloon is accelerating upward at 0.02 m/s$^2$.

What is the radius of the balloon?

Weight of air displaced

$$F = ma$$
$$B - w = ma$$
$$\boxed{\rho V g} - mg = ma$$
$$(1.25)\,(4/3)\,\pi r^3\,(9.8) - 500\,(9.8) = 500\,(0.02)$$

$$r = 4.57 \text{ m}$$

# Pascal's Principle

*"Pressure added to any part of a completely enclosed fluid is added to <u>all</u> parts of the fluid."*

The figure below shows a tank filled with water; at the top is a moveable "piston" fitted snugly into an opening at the top of the tank.

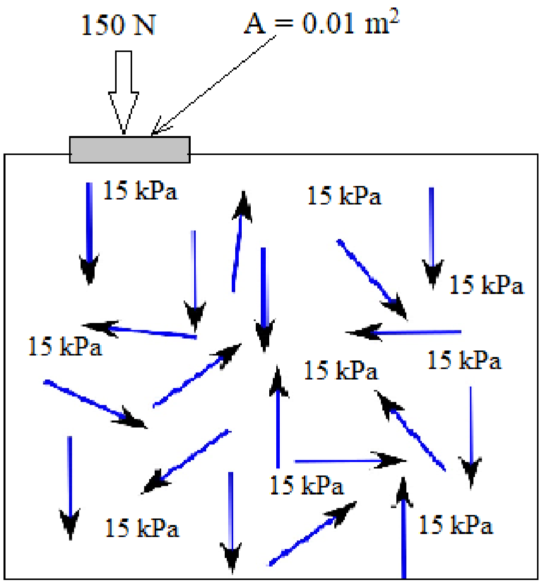
Added Pressure: $F/A = 150\ N/0.01\ m^2$

$= 15{,}000\ Pa$

$= 15\ kPa$

The pressure at each point inside the tank is increased by 15 kPa. Only this added pressure is shown in the figure; pressures that existed before the force was applied are not included.


Scientist, Philosopher, Theologian



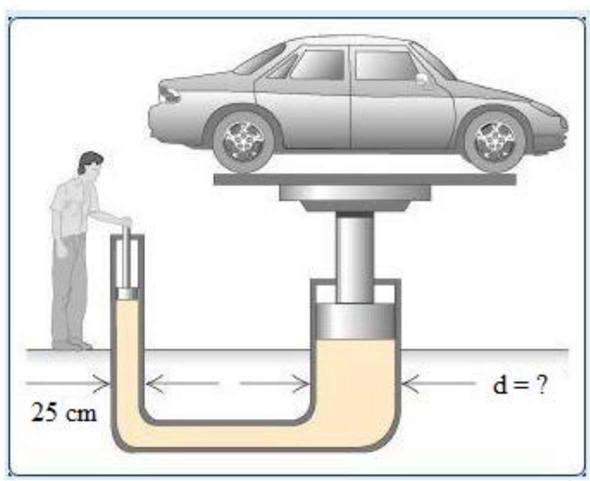
# Pascal's Principle Application

The Hydraulic Lift

Example:

The automobile in the figure weighs 10,000 N (2200 lbs). The push piston has a diameter of 25 cm (10 inches).

What must be the diameter d of the lift piston in order that a 100 N (22 lbs) push by the person will lift the automobile?




Push Piston Area = $\frac{1}{4} \pi (0.25 \text{ m})^2$
$= 0.0491 \text{ m}^2$

Added Pressure $= 100 \text{ N} / 0.0491 \text{ m}^2$
$= 2037 \text{ N/m}^2$

Lift Piston Area = $10{,}000 \text{ N } 2037 \text{ N/m}^2$
$= 4.91 \text{ m}^2$

$(\frac{1}{4} \pi d^2) = 4.91 \text{ m}^2$
$d = 2.50 \text{ m}$

# The Safecracker



Robert DeNiro and Edward Norton co-starred in the movie, “The Score,” in which a safe-cracker (DeNiro) explains to his accomplice (Norton) the method he plans to use in cracking a certain safe is “just simple physics.”

A tiny hole is drilled in the top of the safe; the safe is filled with water through the hole, and a blasting-cap size explosive connected to detonating wires is lowered through the hole into the safe. The creates an omni-directional pressure that’s transmitted to all parts of water and the walls inside the safe. As a result, the walls of the safe are blown outward.

The television program, “Myth Busters,” ran for fourteen seasons; it focused on popular beliefs, Internet rumors, and mechanical effects as portrayed in live-action films. One such focus was on the DeNiro-Norton safecracking..

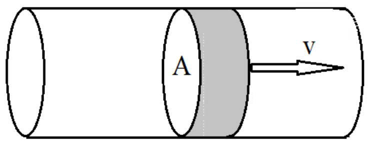
Click link below to view the episode.

Video: MythBusters Test "The Score"

# Volume Flow Rate

The rate at which a fluid travels along a pipe of cross-sectional area A and speed v is measured in cubic meters per second ($m^3/s$) and is symbolized R:

$$R = Av$$




Units: $[m^2]\ [m/s] = m^3/s$

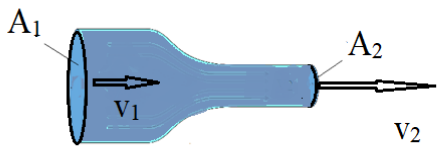
<u>Example :</u>

An empty swimming pool's volume is 64 $m^3$. A pipe filling the pool has an inside radius of 4.0 cm. Water exiting the pipe has a speed of 2.0 m/s. How long will it take to fill the pool?

$$\begin{aligned} R &= Av \\ &= \pi(0.04\ m)^2\ (2.0\ m/s) \\ &= 0.0101\ m^3/s \end{aligned}$$

$$\begin{aligned} t &= (64\ m^3) / (0.0101\ m^3/s) \\ &= 6337\ s \\ &= \text{One hour and 46 minutes} \end{aligned}$$

# Volume Flow Rate Conservation Equation




The figure above shows a portion of a tapered pipe through which water is flowing. The fluid has nowhere else to go except forward, so the flow rate is the same through every cross-section. For example, if 3.0 cubic meters of water pass the left cross section each second, then 3.0 cubic meters of water will have to pass through the right end each second.

We apply this rule to the left and right ends of the pipe above to obtain "The Volume Flow Rate Conservation Equation":

$$R_2 = R_1$$

$$A_2v_2 = A_1v_1$$

<u>Example A</u>:

$A_1 = 0.04\ m^2$
$v_1 = 2.0$ m/s

$v_2 = 8.0$ m/s
$A_2 = ?$

$$\begin{aligned} A_2v_2 &= A_1v_1 \\ A_2 &= (v_1/v_2)\ A_1 \\ &= (2.0/8.0)\ 0.04 \\ &= 0.01\ m^2 \end{aligned}$$

<u>Example B</u>:

$$\begin{aligned} A_2v_2 &= A_1v_1 \\ \pi r_2^2\ v_2 &= \pi r_1^2\ v_1 \\ (r_2/r_1)^2 &= v_1/v_2 \\ r_2/r_1 &= (v_1/v_2)^{1/2} \end{aligned}$$

Assume $r_2$ is the larger of the two radii. How many times greater than $r_1$ must $r_2$ be in order that $v_1$ be 70 times greater than $v_2$?

$$\begin{aligned} (r_2/r_1) &= 70^{1/2} \\ &= 8.37 \\ r_2 &= 8.37\ r_1 \end{aligned}$$

# Garden Hose Flow

If the thumb partially blocks the flow of water from the hose, the flow is tapered across a smaller area. To conserve the volume flow rate, the speed will increase.

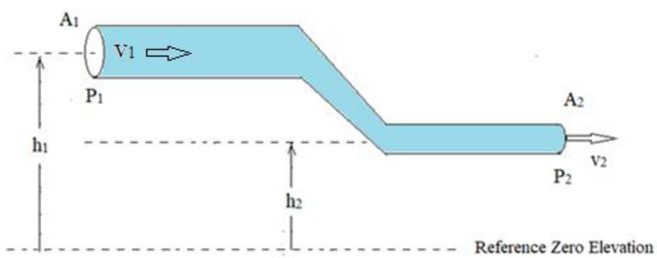
A nozzle accomplishes the same thing as a thumb, but is more efficient.

# Bernoulli's Equation

Video Lecture: Introduction to Bernoulli's Equation

Video Lecture: Bernoulli's Equation Problem

The figure below shows a pipe of varying radius and altitude; only a portion of the pipe is shown.




Bernoulli's Equation

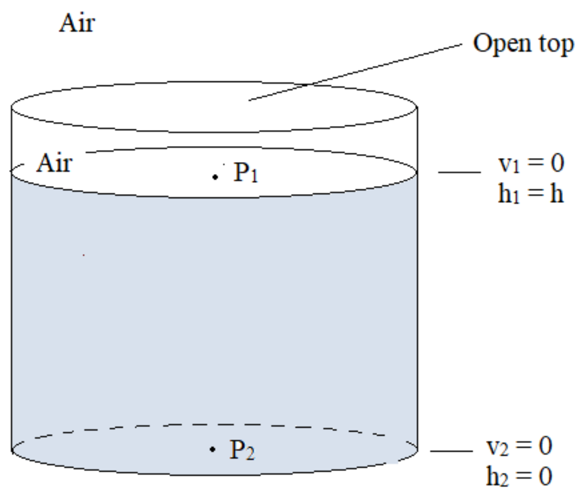
$P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 = P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1$

A second equation that's often used in conjunction with Bernoulli's Equation is the Flow Rate Conservation Equation we discussed previously:

$$A_2 v_2 = A_1 v_1$$

Example:

Water fills a tank to a height h.  Use Bernoulli's equation to find the absolute (total) fluid pressure at the bottom.




The water is stationary, so the speeds at either end are zero. The top is open, so the pressure at the top is atmospheric pressure:

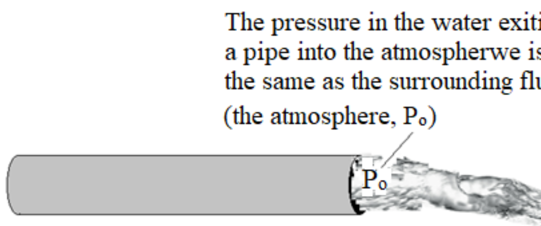
$v_1 = 0$
$v_2 = 0$
$P_1 = P_o$

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1$$
$$P_2 + 0 + 0 = P_o + 0 + \rho g h$$
$$P_2 = P_o + \rho g h$$

The pressure at the bottom of the tank is the sum of the two fluid pressures-- atmospheric pressure, and water pressure-- as expected.

## Pressure in Water Streams in Air

In some Bernoulli's Equation problems, one end of the pipe is open to the atmosphere. The discussion below addresses that situation.




Why?

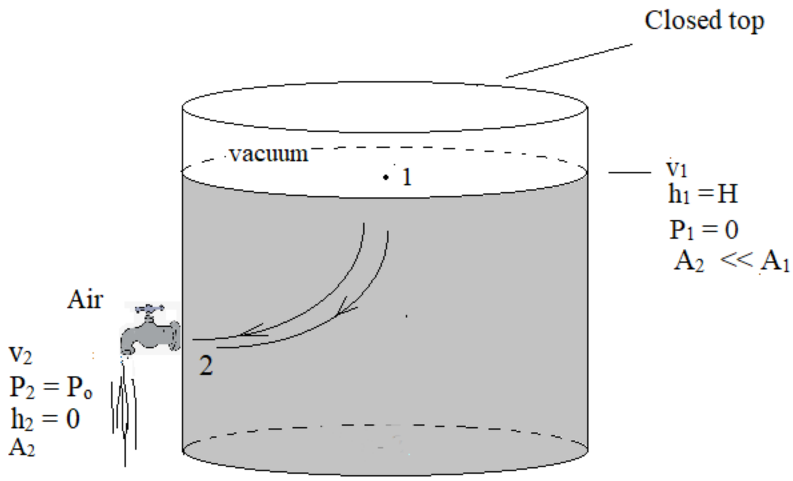
If the water pressure were less than $P_o$, for example, then the atmosphere would "squeeze" the water stream until the water pressure equals the atmospheric pressure.

*In the many examples that follow, if water flows out of the end of the pipe into the atmosphere, this fact will be made explicit. Otherwise, students should assume that either end of the pipe is connected to more pipe.*

Example:

A "wide" tank, *closed* at the top, was initially filled to the top with water. Water ($\rho$ = 1000 kg/m$^3$) is draining out of a faucet. Obtain the speed of the water exiting the faucet in terms of the height H of the water above the faucet.

Note: "Wide" implies that the width of the tank is much larger than the diameter of the faucet.




Recall the volume conservation equation:

$$A_1 v_1 = A_2 v_2$$
$$v_1 = (A_2/A_1)\, v_2$$
$$v_1 << v_2$$
$$v_1 \cong 0$$

The water exiting the faucet is flowing in the atmosphere: ($P_2 = P_o$)

We choose the faucet at Point 2 as the reference elevation:

$$h_2 = 0 \text{ and } h_1 = H$$

The pressure in the vacuum at Point 1 is zero:

$$P_1 = 0$$

Bernoulli's Equation:

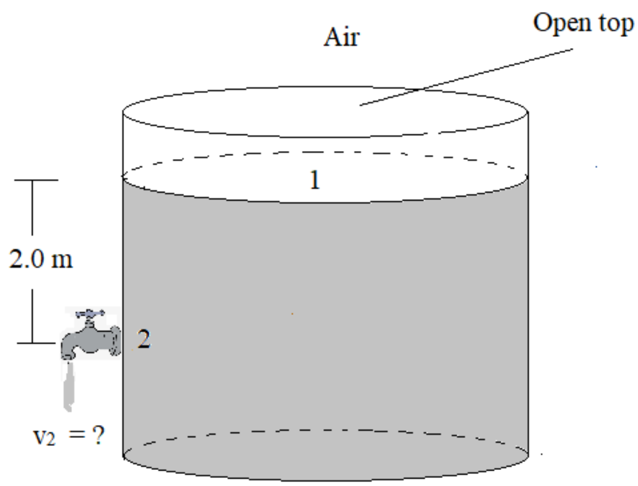
$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h$$

$$101{,}000 + \tfrac{1}{2}(1000)v_2^2 + (1000)(9.8)(0) = 0 + \tfrac{1}{2}(1000)(0)^2 + (1000)(9.8)\,H$$

$$v_2^2 = 9800\,H - 101{,}000$$
$$v_2 = (9{,}800\,H - 101{,}000)^{1/2}$$

Example:

Water is being drained out of a faucet at the bottom of a wide tank of water, open at the top. The faucet is 2.0 meters below the top of the water. What is the speed of the water exiting the faucet?




The area $A_1$ of the top of the tank is much greater than the faucet area $A_2$ through which the water is draining:

$$(A_2/A_1) << 1$$

Applying the Flow Rate Conservation Equation:

$$A_1 v_1 = A_2 v_2$$
$$v_1 = (A_2/A_1)\, v_2$$
$$\cong 0$$

Let the location of faucet be the reference elevation zero:

$$h_1 = 2.0 \text{ m}$$
$$h_2 = 0$$

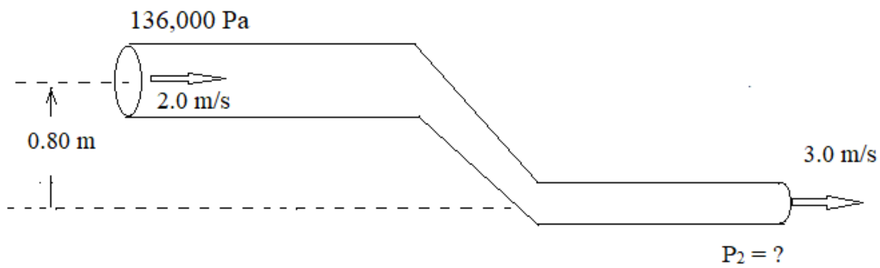
Points 1 and 2 are open to the atmosphere, so

$$P_1 = P_o$$
$$P_2 = P_o$$

$$P_2 + ½ \rho v_2^2 + \rho g h_2 = P_1 + ½ \rho v_1^2 + \rho g h_1$$
$$P_o + ½ (1000) v_2^2 + 0 = P_o + 0 + 1000\,(9.8)\,(2.0)$$
$$v_2 = 6.26 \text{ m/s}$$

Example:

What is the pressure $P_2$ in the tapered portion of the water pipe below? Assume the fluid is water, whose density is 1000 kg/m$^3$.




$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

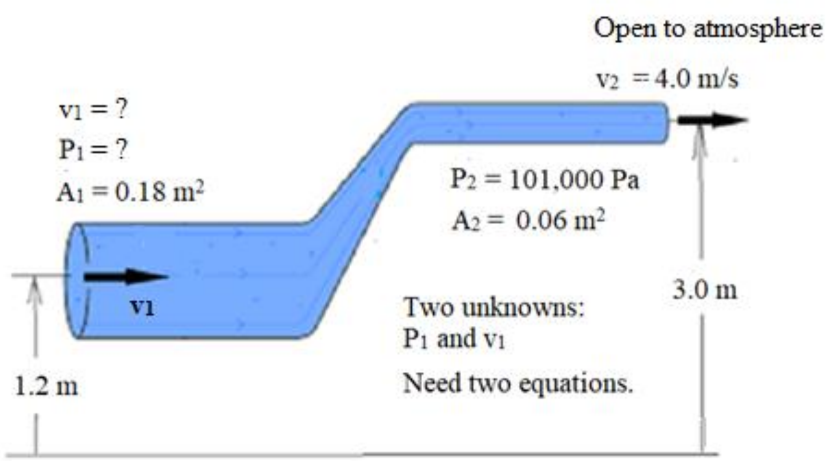
$$136{,}000 + 500\,(2.0)^2 + 9800\,(0.80) = P_2 + 500\,(3.0)^2 + 0$$

$$P_2 = 141{,}340 \text{ Pa}$$

$$= 141.340 \text{ kPa}$$

Example:

What is the pressure $P_1$ and speed $v_1$ at the lower end of the pipe below?




The pressure at the right end is $P_o$ because that end interfaces with the atmosphere.

The two equations needed to solve for the two unknowns are The Fluid Conservation Equation and Bernoulli's equation:

Fluid Conservation Equation:

$$A_1v_1 = A_2v_2$$
$$0.18\ v_1 = 0.06\ (4.0)$$
$$v_1 = 1.33\ \text{m/s}$$

Bernoulli's Equation:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_1 + \tfrac{1}{2}(1000)(1.33)^2 + (1000)(9.8)(1.2) = 101{,}000 + \tfrac{1}{2}(1000)(4.0)^2 + (1000)(9.8)(3.0)$$

$$P_1 = 125{,}755\ \text{Pa}$$
$$= 125.755\ \text{kPa}$$