Physics 23 Chapter 10

Oscillatory Motion

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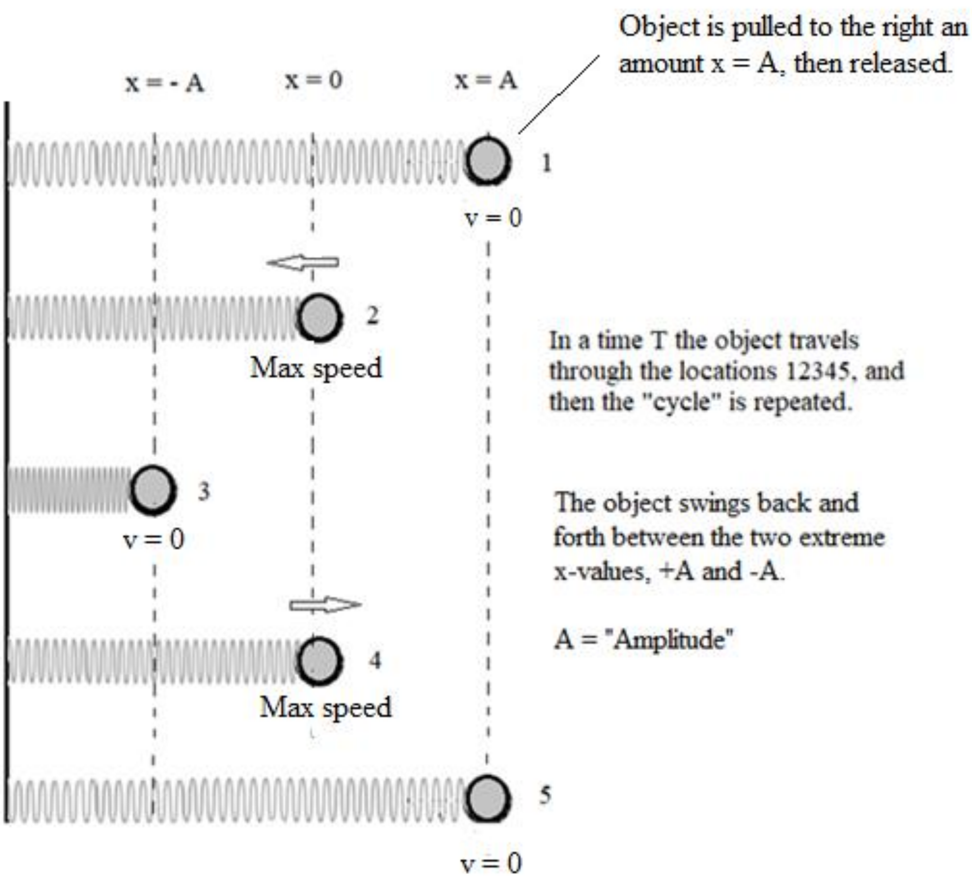
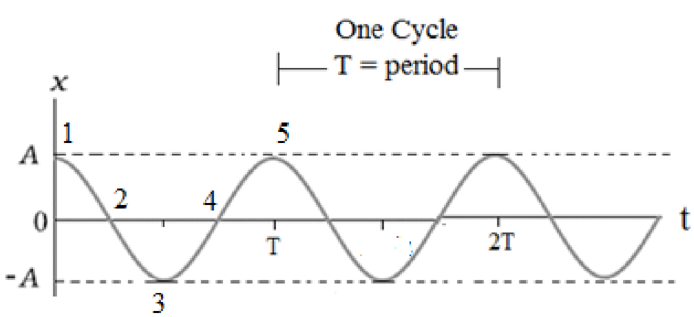
An object is oscillating along the x-axis at the end of a spring. As the system oscillates, the x-coordinate of the object varies between the two extremes x = +A, and x = -A. These are the "turn-around" points, the places where the object is momentarily at rest.



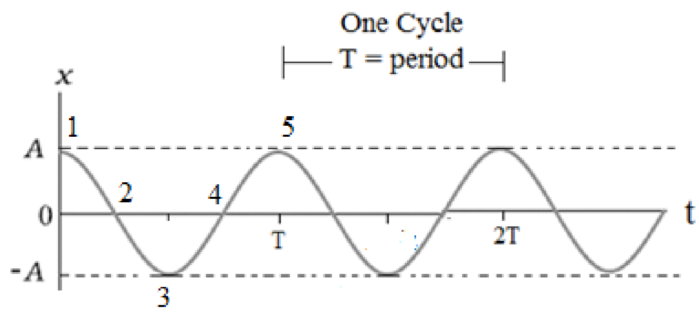
The graph below represents the oscillatory motion above.



T = "Period" of the motion. It's the time it takes to complete one cycle of motion.

f = 1/T
= Number of cycles per second
= Frequency of oscillation

The graph on the previous page is repeated below:



The graph above corresponds to the equation below:

$$x = A \cos[(2\pi/T)t]$$

To simplify the equation of motion, we define a quantity called "angular frequency":

$$\omega = 2\pi/T \quad \text{or}$$
$$\omega = 2\pi f$$

The units of angular frequency are rad/s.*

$$x = A \cos(\omega t)$$

The units of the argument (ωt) of the cosine function are "radians."

Without proof: $\omega = (k/m)^{1/2}$

*Note that the units of angular frequency (rad/s) are the same as the units of angular *velocity*, a quantity discussed in the chapter dealing with rotational motion, and the symbols for these two quantities are also the same. Nevertheless, these quantities are not related.

Example :

k = 600 N/m
m = 1.50 kg
A = 0.40 m

(a) What is the equation of motion?

$\omega = (k/m)^{1/2}$
$= (600/1.5)^{1/2}$
$= 20$ rad/s (or, 20 s^{-1})
$x = A \cos(\omega t)$
$x = 0.40 \cos(20\ t)$

(b) What is the period of motion?

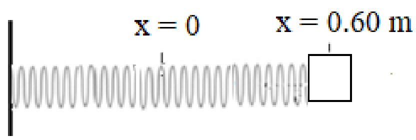
$\omega = 2\pi/T$
$20\ s^{-1} = 2\pi/T$
$T = 0.31$ s

(c) What is the frequency of oscillation?

$f = 1/T$
$= 1/0.31$ s
$= 3.23\ s^{-1}$
$= 3.23$ Hz

Example:

The object in the figure below is pulled 0.60 m to the right and released at time t = 0 s.



$A = -0.60$ m
$k = 288$ N/m
$m = 2.0$ kg
$\omega = (k/m)^{1/2}$
$= 12.0$ rad/s

What will be the object's x-coordinate when t = 0.64 s?

$x = A \cos (\omega t)$
$= 0.60 \cos [12.0 (0.64)]$
$= 0.10$ m

Example:

An object is initially at rest at the end of a stretched spring and is released at time t = 0.

At time t = 1.14 seconds the object is at x = 0.22 m. What was its x-coordinate earlier, when t = 0.41 s?

k = 500 N/m
m = 5.0 kg
$\omega = (500/5.0)^{1/2}$
$= 10.0$ rad/s

$x = A \cos(\omega t)$
$0.22 = A \cos[10.0 (1.14)$
$A = 0.56$ m

$x = 0.56 \cos[(10.0 (0.41)]$
$= -0.32$ m

Example:

A spring-mass system is oscillating on a frictionless tabletop. The spring constant is 500 N/m, and the object's mass is 3.47 kg. The object is pulled 0.80 m to the right and released at time t = 0.

(a) What is the system's initial total energy?

$E_o = K_o + U_o$
$= 0 + ½ (500)(0.80)^2$
$= 160$ J

(b) At what x-coordinate will K = (2/3) U?

160 J = 2/3 U + U
= 5/3 U
$= 5/3 (½ kx^2)$
$= 5/3 [½ (500 \text{ N/m}) x^2]$
x = 0.62 m

(c) What is the earliest time (in milli-seconds) when x = 0.62 m?

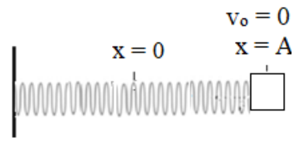
$\omega = (500 \text{ N/m}/3.47 \text{ kg})^{1/2}$
$= 12 \text{ s}^{-1}$ (or `12 rad/s)

x = A cos (ωt)
0.62 = 0.80 cos (12 t)

t = 0.057 s
= 57 ms

Example:

The motion of a spring-mass system begins when the object is pulled to the right by an amount A and released. At that moment, $v_o = 0$ and $x_o = A$.



Obtain an expression for the maximum positive and negative velocities of the object.

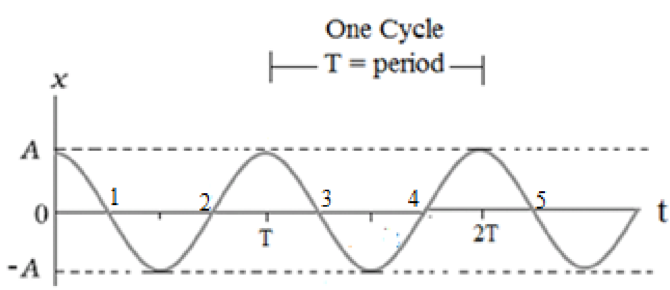
Solution:

$$K + U = K_o + U_o$$
$$\tfrac{1}{2} mv^2 + \tfrac{1}{2} kx^2 = \tfrac{1}{2} mv_o^2 + \tfrac{1}{2} kx_o^2$$
$$= 0 + \tfrac{1}{2} kA^2$$

$$\tfrac{1}{2} mv^2 = \tfrac{1}{2} kA^2 - \tfrac{1}{2} kx^2$$

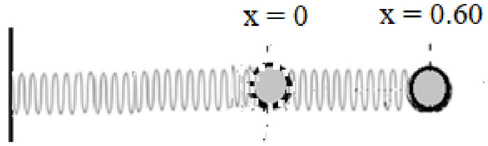
Maximum positive and negative velocities occur when the right side of the equation above is maximum, which occurs when $x = 0$:

$$\tfrac{1}{2} mv^2 = \tfrac{1}{2} kA^2$$
$$v = \pm (k/m)^{1/2} A$$
$$= \pm \omega A$$



The largest positive velocities occur when the object is moving to the *right* through $x = 0$, such as at points 2 and 4. Maximum negative velocities occur at points 1, 3, and 5, when the object is moving to the *left*.

Example:



The spring constant of a spring resting on a tabletop is 1200 N/m. The left end is attached to a wall, and the right end is attached to an object whose mass is 20 kg.

The object is pulled 0.60 m to the right and released at time t = 0.

(a) What are the angular frequency ω, amplitude A, equation of motion, and total energy of the resulting oscillation?

$$\omega = (k/m)^{1/2}$$
$$= (1200/20)^{1/2}$$
$$= 7.75 \text{ rad/s}$$

$$A = 0.60 \text{ m}$$

$$x = 0.60 \cos (7.75\, t)$$

$$E_o = \tfrac{1}{2} kA^2$$
$$= \tfrac{1}{2} (1200)(0.60^2)$$
$$= 216 \text{ J}$$

(b) What is the first time at which the spring is compressed by 0.20 m? (MODE: RADIAN)

$$x = A \cos (\omega t)$$
$$-0.20 = 0.60 \cos (7.75\, t)$$
$$t = 0.25 \text{ s}$$

(c) What is the object's speed at that moment?

$$\tfrac{1}{2} (20)\, v^2 + \tfrac{1}{2} (1200)\, (0.20^2) = 216$$
$$v = \pm 4.38 \text{ m/s}$$

speed = 4.38 m/s

Summary of Important Equations

All of the example problems in this chapter can be solved using only the basic ideas found in the equations below:

$$x = A \cos(\omega t)$$

$$\omega = (k/m)^{1/2}$$

$$\omega = 2\pi/T$$

$$K = \tfrac{1}{2} mv^2$$

$$U = \tfrac{1}{2} kx^2$$

$$E = K + U$$

$$E = E_o$$