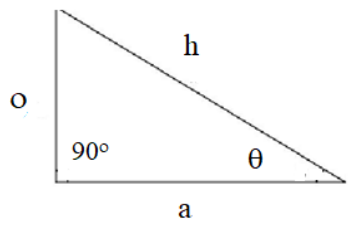
Physics 23 Chapter 1

Dr. Joseph F. Alward

Elementary Trigonometry and Vectors



Mnemonic:
SOH-CAH-TOA

$\sin\theta = o/h$ (OH)
(opposite over hypotenuse)

$\cos\theta = a/h$ (AH)
(adjacent over hypotenuse)

$\tan\theta = o/a$ (OA)
(opposite over adjacent)

$\tan\theta = \sin\theta / \cos\theta$

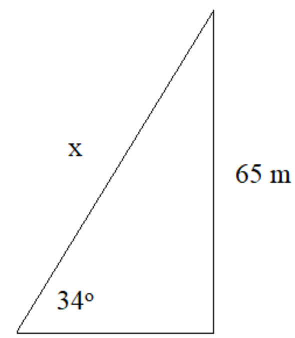
$\sin\theta = \cos(90 - \theta)$
$\cos\theta = \sin(90 - \theta)$
(The sine of an angle equals the cosine of its complement, and vice-versa.)

$\sin 50 = \cos 40$
$\cos 20 = \sin 70$

$a^2 + o^2 = h^2$ (Pythagorean Theorem)

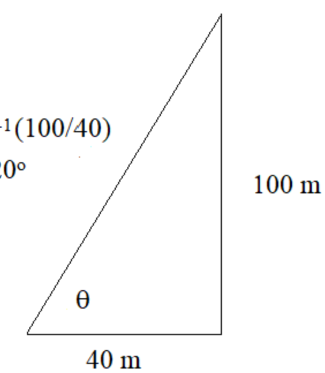
<u>Examples</u>:

(Make sure you calculator is set to “Degree,” not “Radian.”)



$65/x = \sin 34$
$x = 116.24$ m

$\theta = \mathrm{Tan}^{-1}(100/40)$
$= 68.20°$



Length, Distance, and Time Units

Lengths and distances in this course will most commonly be measured in terms of "meters" (m). The meter is the "standard international" unit of length.

yardstick

meterstick

1.0 meter ≅ 3.28 feet (39.4 inches)

Useful conversion factors:

5280 feet /mile
3.28 feet/meter

The "second" is the time unit most commonly used in this course; it's the SI unit for time.

Useful conversion factors:

60 s/min
3600 s/hour

Scalars

Scalars are numbers which tell "how much" of a quantity there is and consist of a number and a unit. For example, the length of a table is 2.40 m; the time it takes a stone to fall to the ground, 1.65 s. These are scalar quantities.

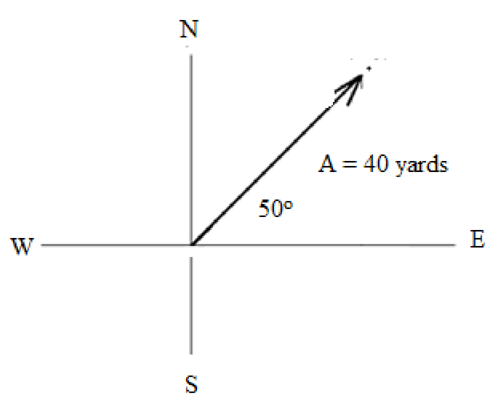
Displacement Vectors

"Vectors" are quantities that have the two scalar attributes (number + units), plus a third attribute: direction. An automobile moving at 25 meters per second, northward. A 100-lb force directed to the right.

The numerical part of a vector, including the units, is called the "magnitude" of the vector. Vectors are commonly displayed as arrows, and lengths of these arrows are proportional to magnitude. For example, if a five-inch arrow drawn on a whiteboard represents a 40-lb force, then an 80-lb force would be an arrow ten inches long. Another name for "magnitude" is "length." Vectors have lengths and direction

A common example of a vector is "displacement." A displacement occurs whenever an object travels a certain distance along some direction from one place to another place.

An example of a displacement is given below an object moves 40 yards along a direction "50 degrees north of east." The "magnitude" of the displacement (the distance moved) is symbolized as "A."



Headings

“Headings” are the directions relative to north, in degrees clockwise, along which some object is moving, or some force is pointing, for example.

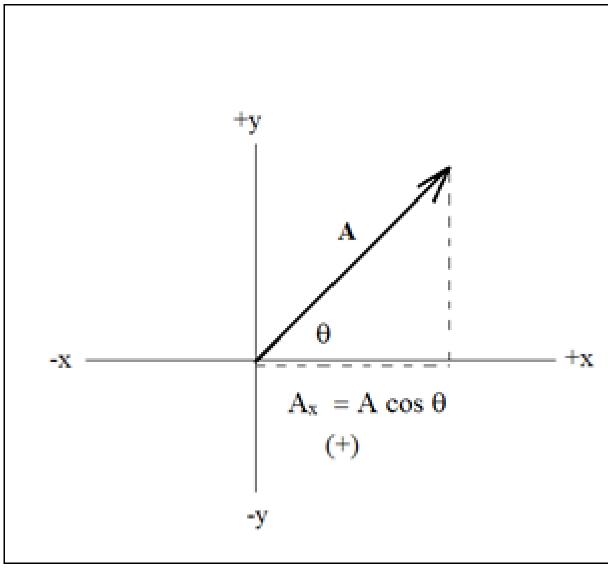
Heading Examples:

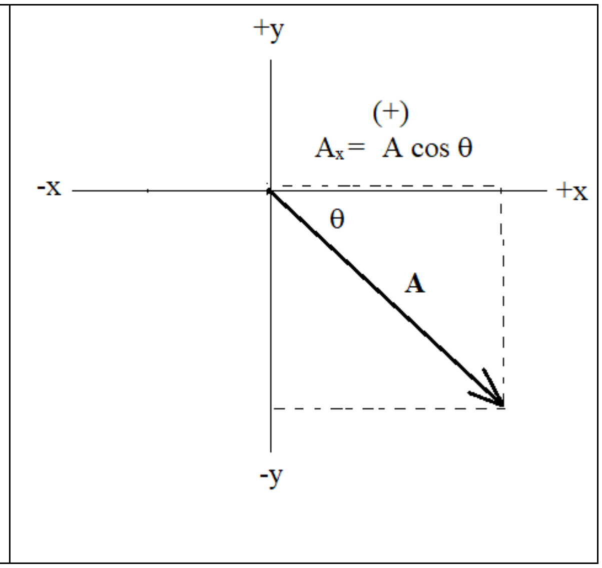
east	90^o
south	180^o
west	270^o
north	0^o, 360^o
northeast	45^o
southeast	135^o
southwest	225^o
northwest	315^o
40^o south of east	130^o
60^o west of north	300^o
15^o east of south	165^o
20^o north of east	70^o

X-Components of Vectors

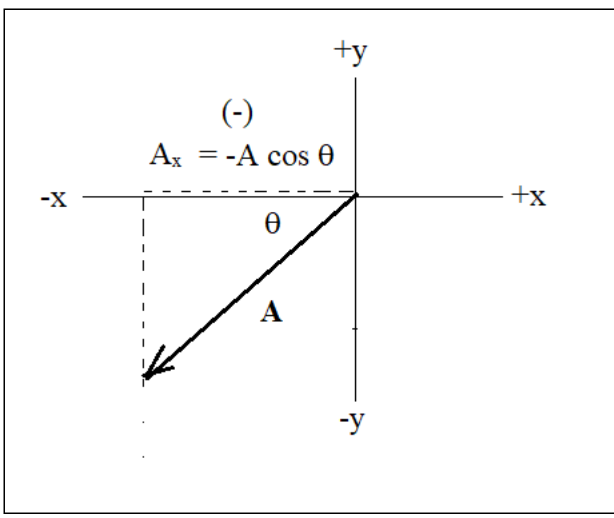
The four rules presented below assume the angle θ is the angle the vector makes relative to the x-axis.

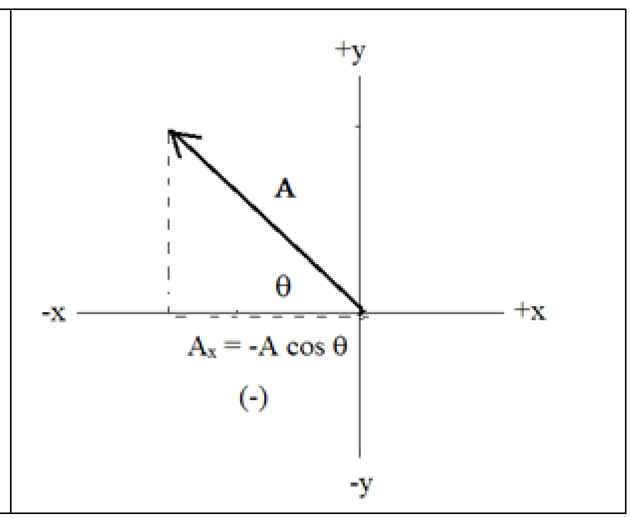
A_x is <u>positive</u> if the vector points into either one of the two <u>right</u> quadrants:





A_x is <u>negative</u> if the vector points into either one of the two <u>left</u> quadrants:

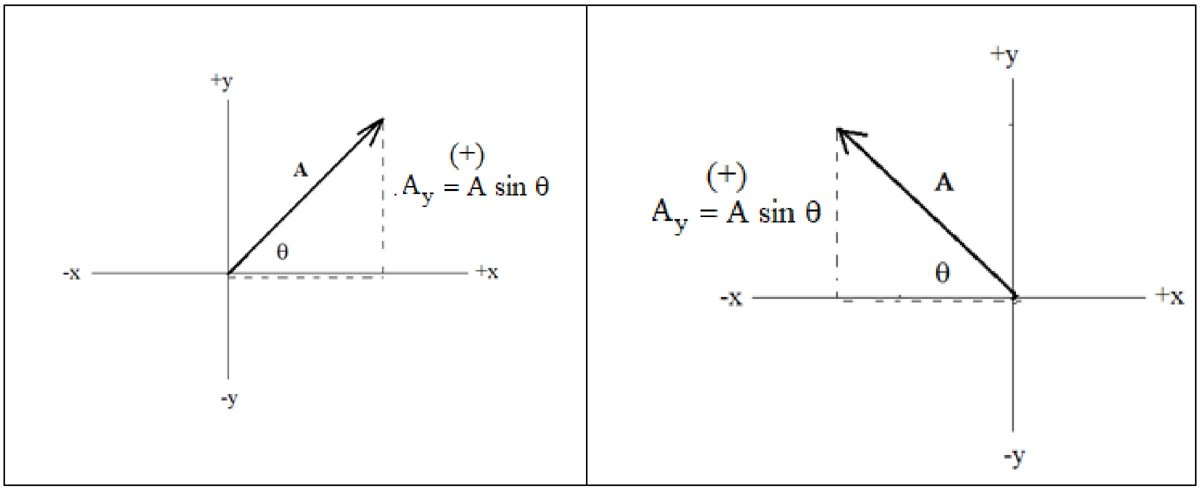




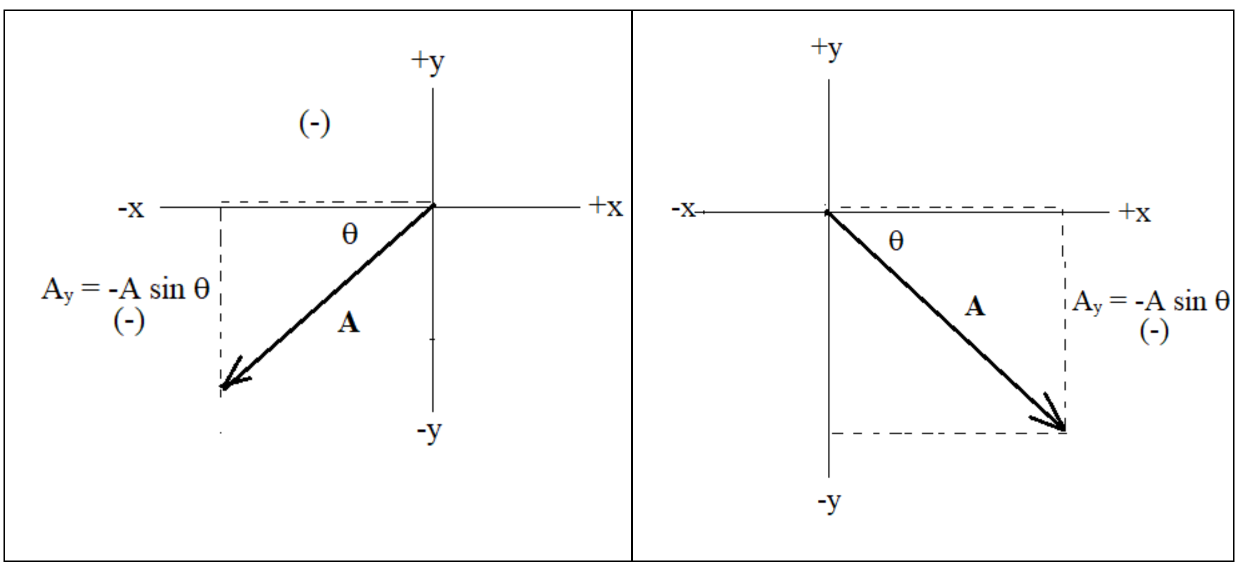
Y-Components of Vectors

The four rules presented below assume the angle θ is the angle the vector makes relative to the x-axis.

A_y is <u>positive</u> if the vector points into either one of the two <u>upper</u> quadrants:



A_y is <u>negative</u> if the vector points into either one of the two <u>lower</u> quadrants:



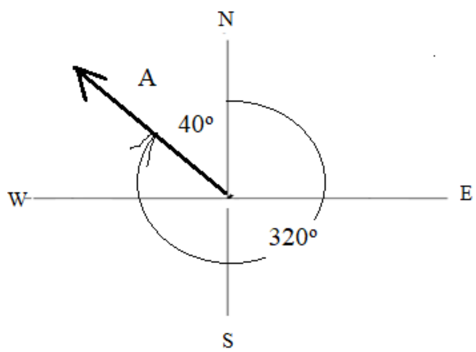
Mnemonic for x-components: the word "right" has a positive connotation, while "left" has a negative connotation.

Mnemonic for y-components: the word “upper” has a positive connotation, while “lower” has a negative connotation.

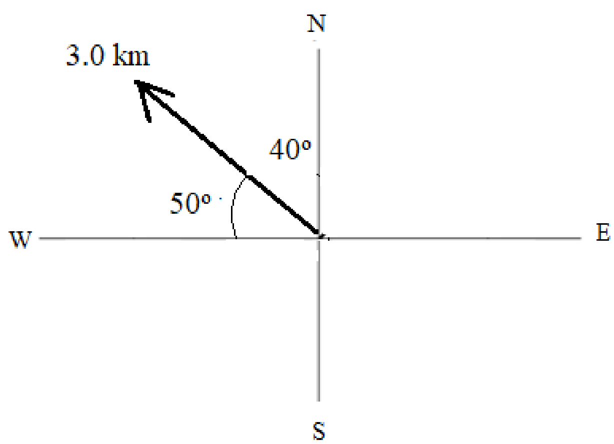
Example :

Note: 1 kilometer (km) = 1000 m

A displacement of 3.0 kilometers occurs along a heading of 320°. What are this displacement’s components?



The angle the vector makes relative to the x-axis is $90^o - 40^o = 50^o$.



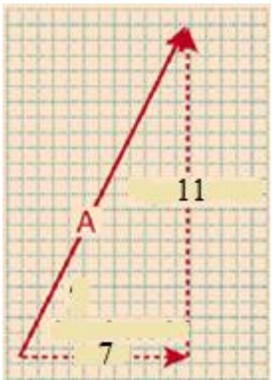
left, and upper rules:

$A_x = -3.0 \cos 50$ (negative because vector points to left quadrant)
$= -1.93$ km

$A_y = 3.0 \sin 50$ (positive because vector points to upper quadrant)
$= 2.30$ km

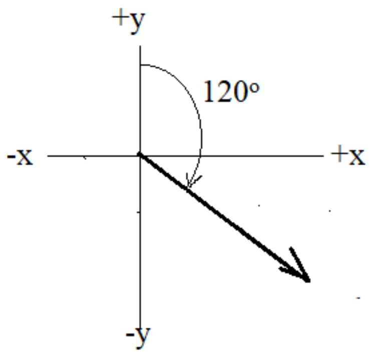
Calculate the Magnitude of a Displacement Vector

Given the components of a displacement vector, how is its length (magnitude) determined?

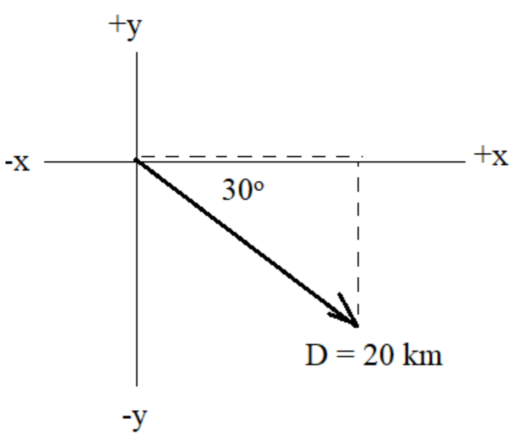
<u>Example:</u>  A_x = 7 km A_y = 11 km A = ?	Apply the Pythagorean Theorem: The length A is the hypotenuse of the right triangle whose side lengths are A_x and A_y. $A = (A_x^2 + A_y^2)^{1/2}$ $= (7^2 + 11^2)^{1/2}$ $= 13.0$

Example:

A displacement **D** has a length of 20 km along a heading of 120°. What are D_x and D_y?



Subtract: 120 – 90 = 30°



D_x = 20 cos 30
= 17.32 km
D_y = -20 sin 30
= -10.00 km

Adding Displacement Vectors

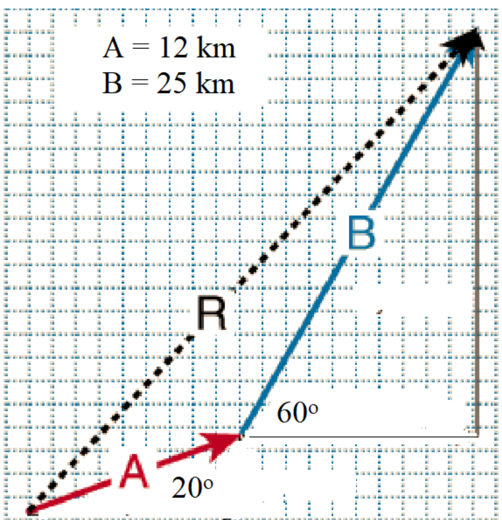
	Video Lecture 3: Vector Addition Part 1 Video Lecture 4: Vector Addition Part 2

Example:

One part of a hiker's journey, labeled **A**, takes her 12 kilometers (km) along a direction that is 20^o north of east (20^o above the +x-axis). The second part, **B,** is 25 km along 60^o north of east (60^o above the +x-axis).

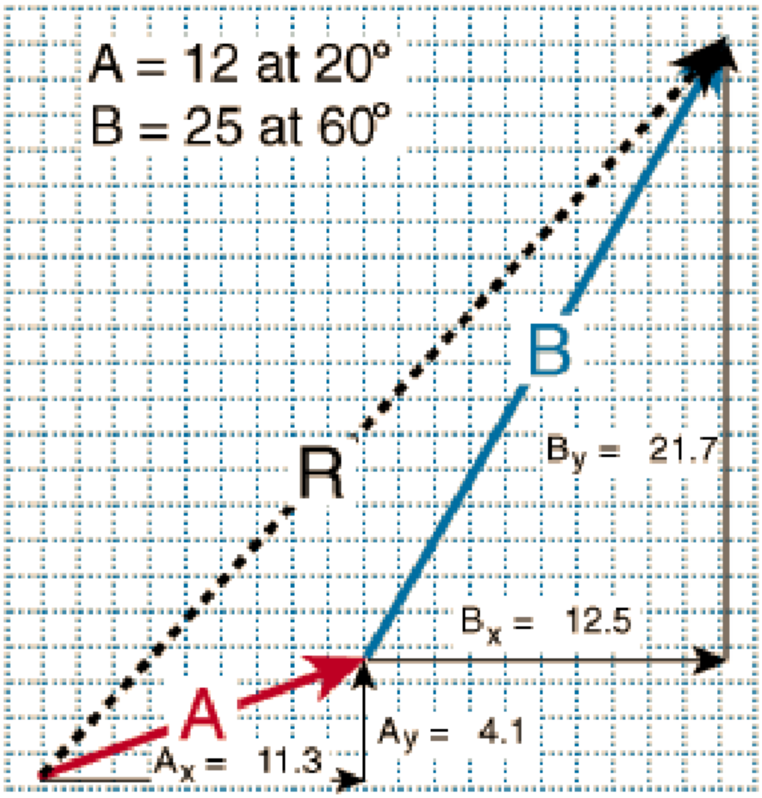
How far from her starting point is she at the end of the hike?

The hiker's displacement is indicated below as **R**.



This problem is continued on the next page.

(Continuation of the hiker's two-part journey problem.)



$R = (R_x^2 + R_y^2)^{1/2}$
$R_x = A_x + B_x$
$= 12 \cos 20^o + 25 \cos 60^o$
$= 11.3 + 12.5$
$= 23.8$ km
$R_y = A_y + B_y$
$= 12 \sin 20 + 25 \sin 60$
$= 4.1 + 21.7$
$= 25.8$ km

$R = (23.8^2 + 25.8^2)^{1/2}$
$= 35.1$ km

(Continued on next page.)

Summary of Vector Addition Rules:

$\mathbf{R} = \mathbf{A} + \mathbf{B}$
$R_x = A_x + B_x$
$R_y = A_y + B_y$
$R = (R_x^2 + R_y^2)^{1/2}$

<u>Example</u>:

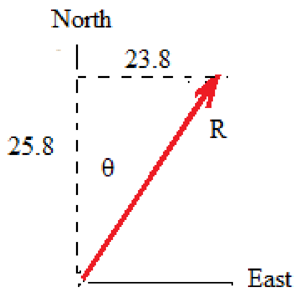
Refer to the hiker displacement example on the previous page.

The results above show that a second hiker, starting her hike at the same point as did the first hiker, would meet the first hiker if she were to travel 35.1 km along the straight-line distance R indicated in the figure.

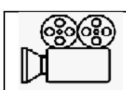
Along what heading θ would that second hiker need to travel from the same starting point to arrive at the first hiker's final location?

From the previous work, we know that the first hiker is 23.8 km east of her starting point, and 25.8 km north. Her displacement is shown below.

Remember: headings are angles measured clockwise relative to the north direction. The heading we're looking for is the angle θ below.



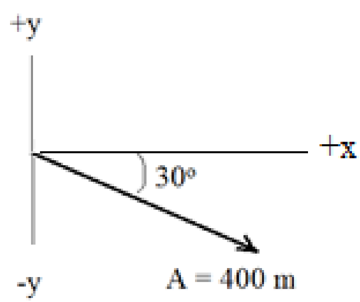
Answer: $\theta = \mathrm{Tan}^{-1}(23.8/25.8)$
$= 42.7^{o}$



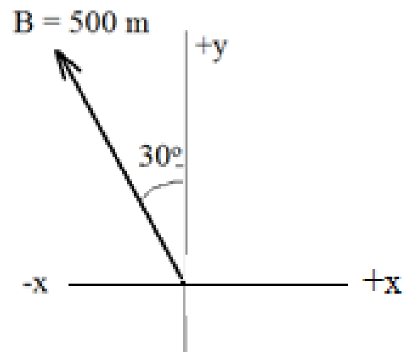
	Video Lecture: Calculating Headings from Components

Example: A boat undergoes two displacements, **A**, and **B**. Along what heading, and how far, could a second boat, starting out at the initial location of the first boat, have to travel to arrive at the first boat's location?

<table>
<tr>
<td>Displacement A: 400 m, heading 120°



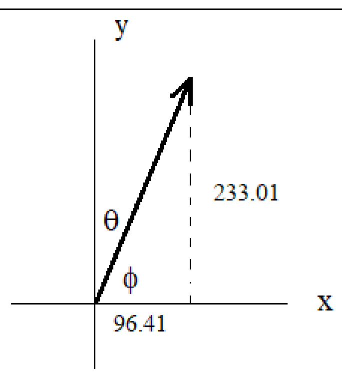
$A_x = 400 \cos 30$
$\quad = 346.41$ m
$A_y = -400 \sin 30$
$\quad = -200.00$ m</td>
<td>Displacement B: 500 m, heading 330°



$B_x = -500 \sin 30$
$\quad = -250.00$ m
$B_y = 500 \cos 30$
$\quad = 433.01$ m</td>
</tr>
</table>

The first boat's displacement (call it **C**) is the sum of the boat's two displacements:
C = A + B

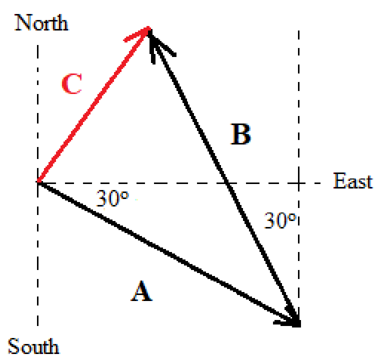
$C_x = A_x + B_x$ $= 346.41 - 250.00$ $= 96.41$ m	$C_y = A_y + B_y$ $= -200.00 + 433.01$ $= 233.01$ m	$C = (C_x^2 + C_y^2)^{1/2}$ $= (96.41^2 + 233.01^2)^{1/2}$ $= 252.17$ m

<table>
<tr>
<td>
</td>
<td>$\phi = \tan^{-1}(233.01/96.41)$
$\theta = 90 - \phi$
$\quad = 22.48°$

The second boat would need to travel 252.17 m along a heading of 22.48°.</td>
</tr>
</table>

Example:

The figure below shows the two-part displacements **A** and **B** of the boat in the previous example, and the single displacement **C** undertaken by the second boat.



The diagram above isn't required to solve that problem. It does, however, illustrate the method of "tail-to-head" vector addition:

To obtain the sum of two vectors, do the following:

1. Draw the first vector (A).

2. Place the tail of the second vector (B) at the head of the first vector (A).

3. Draw a line segment pointing from the tail of the first vector (A) to the head of the second vector (B). The resulting line segment is the sum of the two vectors (C).

The principal value in knowing how to do tail-to-head addition of vectors is being able to estimate the sum of added vectors; this estimation can serve as a check on the results obtained using the component method.

Note from the figure that the approximate heading is estimated to be between 20^o and 25^o, which compares favorably to the actual heading of 22.48^o.